

On macroeconomic values investigation using fuzzy linear regression analysis

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Abstract

The theoretical background for abstract formalization of the vague phenomenon of complex systems is the fuzzy set theory. In the paper, vague data is defined as specialized fuzzy sets - fuzzy numbers and there is described a fuzzy linear regression model as a fuzzy function with fuzzy numbers as vague parameters. To identify the fuzzy coefficients of the model, the genetic algorithm is used. The linear approximation of the vague function together with its possibility area is analytically and graphically expressed. A suitable application is performed in the tasks of the time series fuzzy regression analysis. The time-trend and seasonal cycles including their possibility areas are calculated and expressed. The examples are presented from the economy field, namely the time-development of unemployment, agricultural production and construction respectively between 2009 and 2011 in the Czech Republic. The results are shown in the form of the fuzzy regression models of variables of time series. For the period 2009-2011, the analysis assumptions about seasonal behaviour of variables and the relationship between them were confirmed; in 2010, the system behaved fuzzier and the relationships between the variables were vaguer, that has a lot of causes, from the different elasticity of demand, through state interventions to globalization and transnational impacts.

Keywords: *fuzzy linear regression, vague property, genetic algorithms, construction production, agricultural production, GDP*

1. Introduction

Regression models are used in engineering practice wherever there is a need to reflect more independent variables together with the effects of other unmeasured disturbances and influences. In classical statistical regression, we assume that the relationship between dependent variables and independent variables of the model is well-defined and sharp. Although statistical regression has many applications, problems can occur in the situations in which number of observation is inadequate (small data set), difficulties verifying distribution assumptions exist, vagueness in the relationship between input and output variables exists, the ambiguity of events or degree to which they occur or inaccuracy and distortion introduced by linearization is possible [19].

However, in the real world, it is hampered by the fact that these conditions are more or less non-specific and vague. This is particularly true when modelling complex systems which are difficult to define, difficult to measure or in cases where it is incorporated into the human element.

The theoretical background for abstract formalization of the vague phenomenon of complex systems is the fuzzy set theory. In the paper vague data is defined as specialized fuzzy sets - fuzzy numbers and there is a fuzzy linear regression model as a fuzzy function with fuzzy numbers as vague parameters. The determination of regression model uncertainty using fuzzy approaches does not require meeting the above presumptions of statistical regression.

The application part of the fuzzy regression analysis was performed based on the analysis of the time series development of selected macroeconomic variables which can be of a seasonal character in the national economy. These include indicators of construction production, agricultural production and the rate of unemployment in the Czech Republic from 2009 to 2011.

The choice of macroeconomic variables was based both on their seasonal character and on their interrelationships. While construction and agricultural production grows from spring to autumn, the level of unemployment in the period generally decreases. And the indicators of construction production and agricultural production may evolve in the same period differently, which can have many causes: from different elasticity of demand for construction and agricultural production, through different levels and forms of state intervention in

these segments of the national economy, to the influence of foreign trade and globalization. Along with this form of development of the monitored construction and agricultural production variables the unemployment rate does not always behave completely normal and can, irrespective to the development of construction and agricultural production, have increasing and declining trends. The cause of this phenomenon can be seen, inter alia, in low elasticity of labour supply, strong influence of trade unions and an entire labour system of social security, which altogether distort the labour market. All three monitored variables were subjected to the fuzzy regression analysis of the time series development of 12 measured values per year from 2009 to 2011.

2. Fuzzy Regression Model Identification

2.1. Observed Output Variable y^0 Fuzzification

To define the type of the fuzzy regression model we use the version in which the input variables x are mentioned as crisp numbers and the observed values \tilde{Y}^0 as triangular fuzzy numbers, respectively. Thus, let us consider fuzzy number \tilde{Y}^* as the estimate and fuzzy number \tilde{Y}^0 as the observed value of the model output variable respectively. The fuzziness d_j of the observed fuzzy value \tilde{Y}_j^0 at the step observation j can be determined using the observed values at the step $(j+1)$ and $(j-1)$, respectively (see Fig. 1).

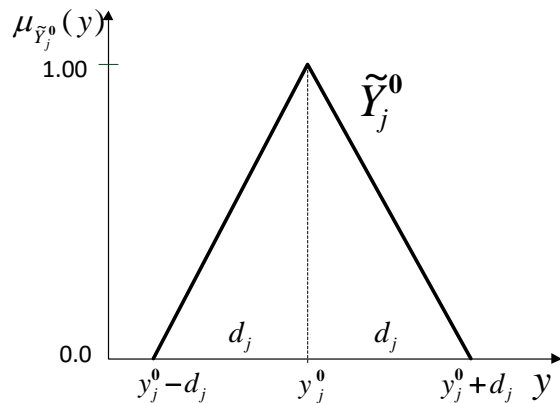


Fig. 1. Triangular membership function of fuzzy numbers \tilde{Y}_j^0
Source: [own processing]

It means, fuzzy number \tilde{Y}_j^0 is mentioned of a unequal triangular type. The values d_j we can calculate by the formulas

$$d_j = \frac{1}{2} |y_{j+1}^0 - y_{j-1}^0| \quad (1)$$

2.2. Fuzzy Regression Coefficients \tilde{A} Determination

Finding values α_i and c_i as searched parameters of fuzzy regression coefficients \tilde{A}_i (Fig.1) is defined as an optimization issue.

Fitness of the linear regression fuzzy model to the given data is measured through the Bass-Kwakernaaks's index H – see Fig.4 [4], [8]. Adequacy of the observed and estimated values is conditioned by the relation (6) – the maximum intersection (consistency) of two fuzzy sets – the estimated \tilde{Y}^* and the examined \tilde{Y}^0 – must be higher than the set value H (see Fig.2).

$$\max_y \{ \mu_{\tilde{Y}^0}(y) \wedge \mu_{\tilde{Y}^*}(y) \} = \text{Cons}(\tilde{Y}^0, \tilde{Y}^*) \geq H \quad (2)$$

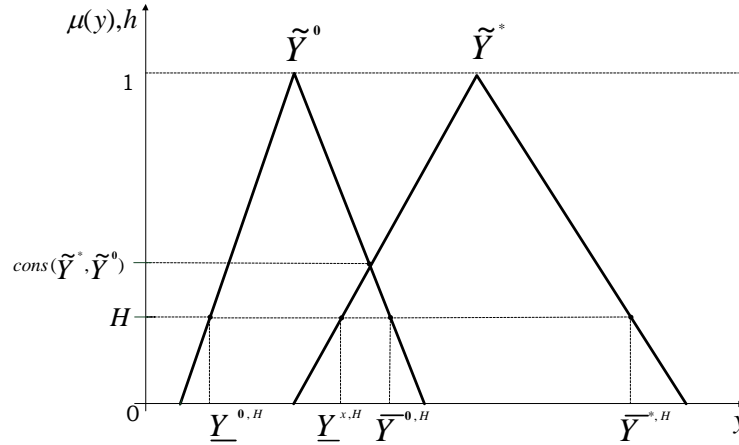


Fig. 2. Adequacy of Linear Regression Model

Source: [own processing]

Only if the condition (6) is fulfilled we assume good estimation \tilde{Y}^* of the observed output value \tilde{Y}^0 .

The relation (6) is satisfied under the condition (see Fig.4)

$$\underline{Y}^{*,H} \leq \bar{Y}^{0,H} \quad (3)$$

$$\underline{Y}^{0,H} \leq \bar{Y}^{*,H} \quad (4)$$

Consider the determined level H the boundary of intervals $Y^{*,H}$ and relations (3), (4) we can express

$$\underline{Y}^{*,H} = -(1-H) \sum_{i=0}^n c_i |x_i| + \sum_{i=0}^n \alpha_i x_i \quad (5)$$

$$\bar{Y}^{*,H} = (1-H) \sum_{i=0}^n c_i |x_i| + \sum_{i=0}^n \alpha_i x_i \quad (6)$$

According to Fig. 2 we can write:

$$\underline{Y}^{*,0} = y^0 + (1-H)d \quad (7)$$

$$\bar{Y}^{*,0} = -y^0 + (1-H)d \quad (8)$$

Consider $j = 1, 2, \dots, m$ observations we can formulate the conditions (7), (8) in final form

$$\sum_{i=0}^n \sum_{j=1}^m \alpha_{i,j} x_{i,j} + (1-H) \sum_{i=0}^n \sum_{j=1}^m c_{i,j} |x_{i,j}| \geq y_j^0 + (1-H)d^0 \quad (9)$$

$$-\sum_{i=0}^n \sum_{j=1}^m \alpha_{i,j} x_{i,j} + (1-H) \sum_{i=0}^n \sum_{j=1}^m c_{i,j} |x_{i,j}| \geq -y_j^0 + (1-H)d^0 \quad (10)$$

$$c_{ij} \geq 0 \quad (11)$$

The requirement on adequacy of the estimated and observed values (6) will be complemented by the requirement on minimum possible total uncertainty of the identified fuzzy regression function

$$\sum_{i=0}^n \sum_{j=1}^m c_{i,j} \rightarrow \min, \quad i = 0, 1, \dots, n, \quad j = 1, 2, \dots, m \quad (12)$$

where $i = 1, 2, \dots, n$ is the number of input values of the regression function and $j = 1, 2, \dots, m$ is the number of observations.

Then we can set the optimization problem

- a) minimization of fuzzy model vagueness
- b) under the condition

To solve the minimization problem under the condition, many authors use the linear programming method [1], [8]. Nevertheless, in this paper we use the genetic algorithm method to solve this problem [16]. Mainly, the reason is that the authors are oriented to use unconventional methods of artificial intelligence in order to prove their quality and efficiency in solving complex tasks. Genetic algorithms are a representative of evolutionary methods; their higher computational complexity is nowadays eliminated by high-performance computing. They are widely used in the search for optimal solutions. They can be well used for the identification of fuzzy regression models where they deal with the task of finding the optimal fuzzy regression coefficients as triangular fuzzy numbers.

The identification of fuzzy regression coefficients – fuzzy numbers $\tilde{A}_0, \tilde{A}_1, \dots, \tilde{A}_n$ – was divided into two tasks

- a) the identification of the mean value (core) α_i of fuzzy number \tilde{A}_i and
- b) the identification of c_i as a half of the width of the carrier bearing $\tilde{A}_i = \{\alpha_i, c_i\}$.

The tasks are solved by using the genetic algorithm in series. First the identification of α_i and then the identification of c_i are done. Thus, the optimization of the fuzzy linear regression model is a two-step process when two genetic algorithms, designated G1 and G2, are used.

For the identification of the mean value (core) α_i of fuzzy number \tilde{A}_i the minimization of the fitness function J_1 is defined in the form

$$\min J_1 = \min \frac{1}{m} \sum_{j=1}^m (y_j^0 - \beta_j)^2 \quad (13)$$

and the genetic algorithm GA1 is used. For the identification of c_i as a half of the width of the carrier bearing \tilde{A}_i the minimization of the fitness function J_2 is defined in the form

$$\min J_2 = \min \sum_{j=1}^m \sum_{i=0}^n |c_{j,i}| \quad (14)$$

and the genetic algorithm GA2 with three constraints is used. Minimization of the fitness function J_2 is based on the previous identification of the role of the mean value (core) α_i and uses the already identified values of α_i for determining the width of the carrier bearing α_i .

The value of $H = 0,5$ is expertly determined in the next part of paper.

3. Time Series Fuzzy Regression Analysis

The fuzzy linear regression model has the opportunity to express not only the analytical linear approximation of multivariate functions, but also the size of its uncertainty (vagueness, fuzziness) in the form of an indeterminate possibility area. The graph of a one-dimensional fuzzy regression function we can see in Fig.5 together with the appropriate linear approximation and the possibility area of the estimated fuzzy output \tilde{Y}^*

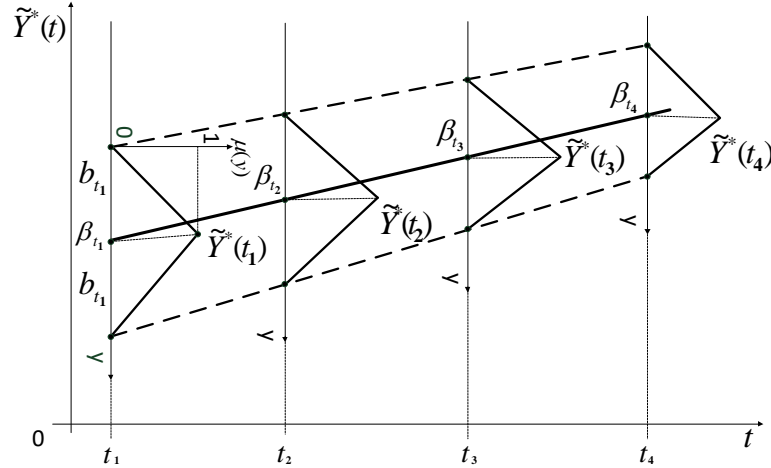


Fig. 3. One-Dimensional Fuzzy Linear Regression Function
Source: [own processing]

The one-dimensional fuzzy time series regression model has the ability to express its trend and seasonal cycles, respectively. Both of these features are enhanced by the possibility area that defines the size of the vagueness of the model and defines the range in which may be the value of the trend and seasonal cycles.

The one-dimensional fuzzy linear regression model of a time series trend is given by the formula

$$\tilde{Y} = \tilde{A}_0 + \tilde{A}_1 t \quad t = 1, 2, \dots \quad (15)$$

The value of a seasonal deviation in every month MSD (as fuzzy number) is calculated for each year $r = 1, 2, \dots, L$ and for each month $k = 1, 2, \dots, 12$ as the difference between the trend value and the actual value to be estimated

$$MSD = (\tilde{Y}_{r,k}^0 - \tilde{Y}_{r,k}^*), \quad r = 1, 2, \dots, L, \quad k = 1, 2, \dots, 12 \quad (16)$$

The central value of fuzzy number MSD is calculated as the difference of the central values $\tilde{Y}_{r,k}^0; \tilde{Y}_{r,k}^*$, the fuzziness is calculated as the sum of fuzziness of fuzzy numbers $\tilde{Y}_{r,k}^0; \tilde{Y}_{r,k}^*$.

The seasonal cycle is then defined as the time series of 12 seasonal deviations for 12 months. A seasonal deviation for a given month $k = 1, 2, \dots, 12$ is calculated as the average value of the month of year $r = 1, 2, \dots, L$ of the considered time series.

$$\tilde{Y}_k^* = \frac{1}{L} \sum_{r=1}^L (\tilde{Y}_{r,k}^0 - \tilde{Y}_{r,k}^*); \quad r = 1, 2, \dots, L, \quad k = 1, 2, \dots, 12 \quad (17)$$

For example, the seasonal variation for the first month of January is calculated as the mean of the January seasonal variations of the considered $r = 3$ years

$$\tilde{Y}_1^* = \frac{1}{3} \sum_{r=1}^3 (\tilde{Y}_{r,1}^0 - \tilde{Y}_{r,1}^*) \quad (18)$$

The values of monthly deviations are calculated as fuzzy numbers. The core of fuzzy number \tilde{Y}_k^* is calculated as the mean difference of the cores, the uncertainty is calculated as the mean of the sum of fuzziness. Thus, we calculate 12 fuzzy numbers, which pass into the timeline of 12 months as a curve of cores and their possibility areas.

4. Selected Economic Variable Investigation

The modelling of economic variables with high degree of uncertainty is very difficult, especially in current times of economic and financial crisis. The development of such variables is subject to a number of influences, both exogenous and endogenous, some of which are in fact hardly predictable or have a prominent degree of fuzzitivity. What is more, the relative effect of non-economic influences upon the development of the selected economic variables is growing in importance, as various subjects on the market – households and companies – contemplate their level of consumption, investment and savings on the basis of their uncertain future. Apart from rational evaluation of the relevant economic data, they are also under pressure of a number of influences from the area of psychology, politics, demographic development, natural circumstances, foreign affairs etc., and the so-called transactional motive is now replaced with the motive of caution. The submitted time series development regression analysis models 12 measured values of the selected variables in the years from 2009 to 2011 under these specifically defined conditions. The analysed time period was selected on the basis of the beginning and the proceedings of the crisis, as 2009 was the first year when the crisis proceeded in its fullest and throughout the year. In the measured period of three years, every year 12 values were analysed, which in the coherent time series already enable a complex, general and valuable analysis.

The selection of variables was methodically chosen with regard to the mutual interconnectivity and collaboration within the mechanism of national economy and their relative importance in the economy. This was the reason why two primary variables from the GDP production area were analysed (construction and agricultural production) and also one secondary variable was analysed (unemployment), which is in causal relationship to the two variables mentioned above. Both construction and agricultural production are variables with a highly seasonal cycle, which is, with some delay, mirrored in the development of unemployment in both directions. Simultaneously with this presupposition, the variables construction and agricultural production, however seasonal, may act differently, which is caused by the obvious differences in the characteristics of these disciplines. Whilst elasticity of demand by agricultural production may be very low, by construction production it is very high, thus households and companies postpone their consumption and investments until after the crisis, so for a more favourable time period. This is why the decrease in production in construction leads to the subsequent increase in unemployment, whilst for agricultural production, this is not the case. For agricultural production is in this case typical the so-called Giffen goods effect, where the demand for agricultural production does not grow nor fall significantly, as the income effect absolutely negates the substitution effect. Therefore also the increase in unemployment along with the decrease of the level of agricultural production is set rather by the effect of the seasonal cycle, whilst in the case of construction production the increase in unemployment is rather dictated by the decrease in demand for construction production.

Selected economic variables (macroeconomic variables) the values of which show a seasonal character (fluctuations) were subjected to the fuzzy regression analysis. Here we talk about construction production, agricultural production and the rate of unemployment) [20].

Construction and agricultural production usually grow from spring to autumn seasons, in winter we can see their decline. The opposite trend is shown by the unemployment rate which drops from spring to autumn, and in winter unemployment generally has the highest values (seasonal unemployment). This is a purely theoretical viewpoint, however, a number of certified work and empirical observations approve it. While the crisis, during the studied values (2009 - 2011), the economy could behave unpredictably. It is interesting to watch positive values of agricultural production, when consumers simply cannot significantly reduce demand, and greater fluctuations including negative values in construction production. However, for both types of production, despite

the mentioned diversity, we can apparently observe seasonal behaviour of variables, especially in the long-term perspective.

Unemployment behaves in the opposite way: when production declines (the GDP), unemployment rises and vice versa (due to fluctuation - decrease/ increase of demand). Thus, unemployment secondarily shows a seasonal character. It is (inter alia) the so called Okun's law which was formulated in the 1960s; it says that when there is a decline in GDP by 2%, the unemployment rate grows by 1%; or the proportion is approximately 2:1.

The identification of the time series fuzzy regression models was made using the standard genetic algorithms of the Optimtoolbox MATLAB program system [9].

The results are shown in the form of the fuzzy regression models of time series of Construction production (CPT) - Fig.4 and Fig. 5, Agricultural production (APT) - Fig. 6 and Fig. 7, and Unemployment (UNT) - Fig. 8 and Fig. 9. The figures represent their fuzzy trends and fuzzy seasonal cycles. Appropriate regression coefficients of regression functions are presented in the form $A\{\alpha; c\}$

$$A_0\{-0.7268; 2.0190\}$$

$$A_1\{-0.1533; 0.1187\}$$

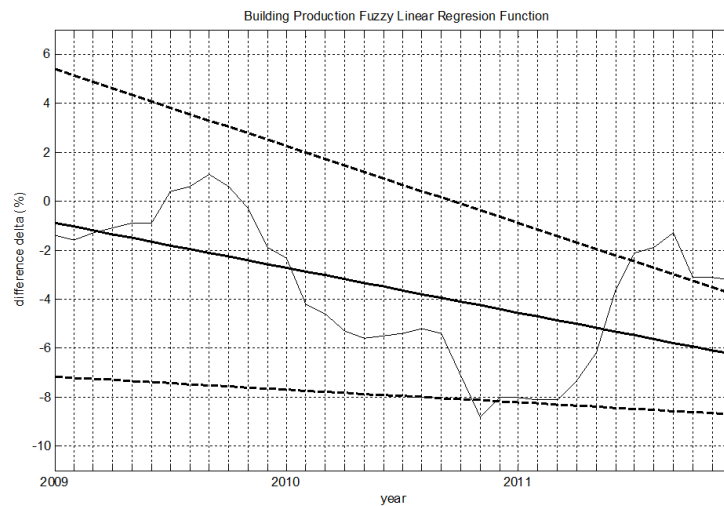


Fig. 4. Construction Production - Fuzzy Linear Regression Function
Source: [own processing]

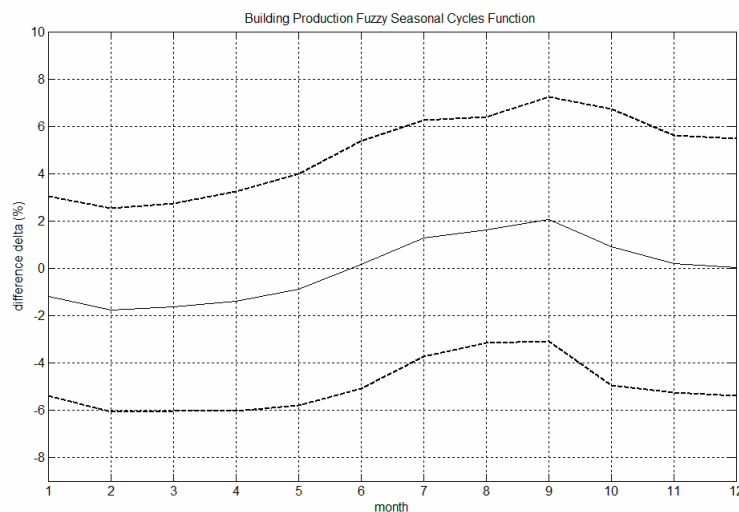


Fig. 5. Construction Production - Fuzzy Seasonal Cycles Function
Source: [own processing]

$$A_0 \{4.9646; 0.7891\}$$

$$A_1 \{0.1157; 0.0243\}$$

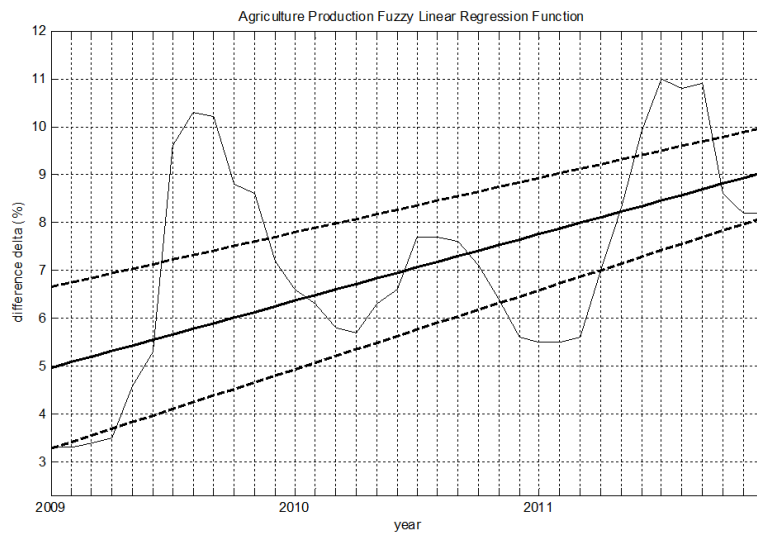


Fig. 6. Agricultural Production - Fuzzy Linear Regression Function
 Source: [own processing]

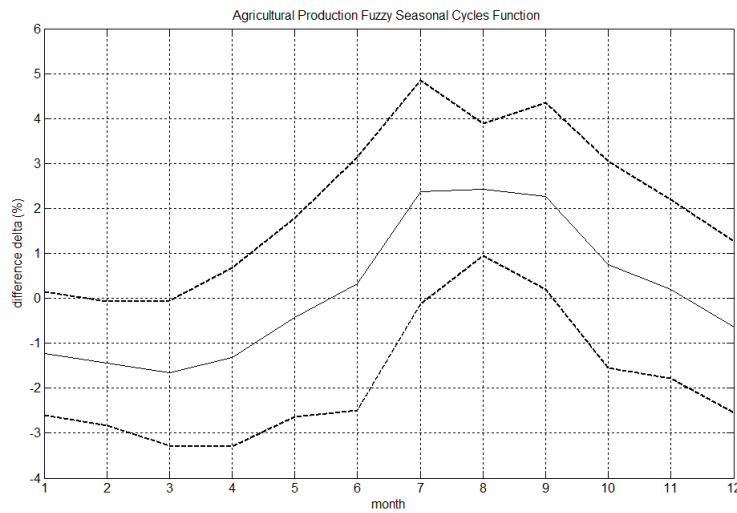


Fig. 7. Agricultural Production - Fuzzy Seasonal Cycles Function
 Source: [own processing]

$$A_0 \{7.9438; 0.5790\}$$

$$A_1 \{-0.0385; 0.0057\}$$

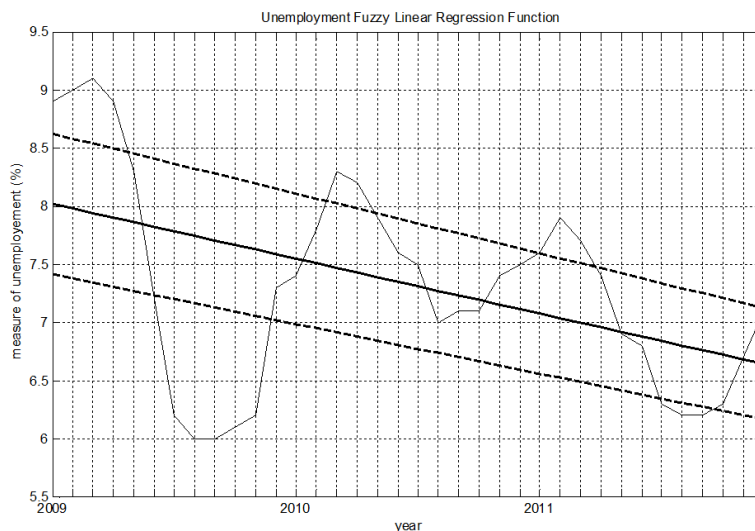


Fig. 8. Measure of Unemployment - Fuzzy Linear Regression Function
Source: [own processing]

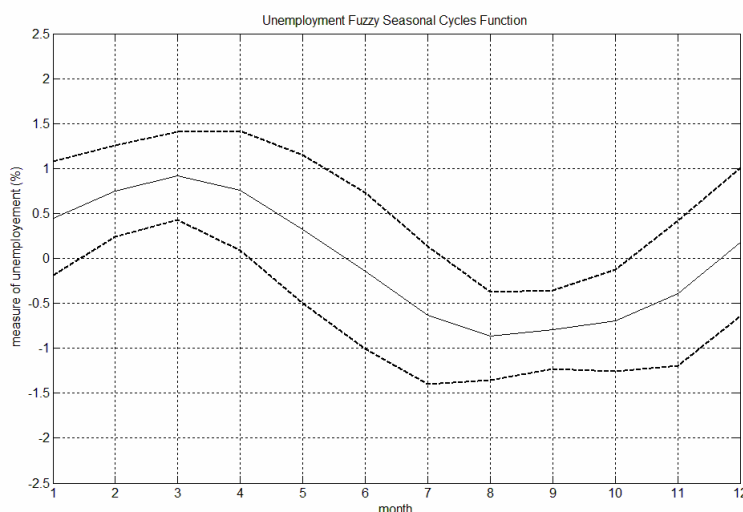


Fig. 9. Measure of Unemployment - Fuzzy Seasonal Cycles Function
Source: [own processing]

The results of the time series analysis of the development in macroeconomic variables of the unemployment rate (UNT), construction (CPT) and agricultural production (APT) which were found above show the interdependence of some of these demonstrated variables, but in some cases they also reflect a certain degree of vagueness, i.e. fuzzitivity. This concerns both the interdependence of CPT and APT variables in relation to UNT, and also in some periods basically the same trend of CPT and APT (2009 and 2011) and their opposite trend in 2010. The dependencies selected above were confirmed by a number of impartial scientific methods and long-term professional empirical observation [2]. However, some of the above mentioned assumptions were not confirmed by the presented work and it has some general reasons that can be satisfactorily explained.

One of the reasons can be seen in the market failure due to the global economic crisis. We rather do not talk about a state of stagnation or moderate inflation, but about the current so-called stagflation, which used to be a relatively rare phenomenon. It is a combination of two failures of macroeconomic equilibrium, namely economic stagnation, or rather stagnation of GDP growth, and rising prices (inflation). The existence of this type of failure raises serious national economic problems having impact on the fiscal and monetary policy of the country with an emphasis on the contradictory nature of these failures; especially the choice of current fiscal expansionary and also restrictive monetary instruments of the economic policy [17]. What also plays its role here is the global type of economy and thus limited effectiveness of measures at the national level. This is especially true for very small and open economy such as the Czech Republic.

Another important influence on the variables UNT, CPT and APT in time is the so-called time lag in the economy. It means series of delays resulting from the characteristics of an economic process based on a premise that from the moment when the problem (failure) appeared and then while watching the problem using conclusive measurable economic tools (recognition lag) some time (delay) will always pass; then there is particular time needed for making a decision and choosing tools for fixing the failure, and also there is time required to implement the tools including their positive effect (implementation lag). This fact significantly reduces efficiency of the economic policy and together with its global character fundamentally affects economic activity [5]. Then it refers to the mutual correlation of all the variables and their existing and proven fuzzitivity.

Another significant circumstance affecting fuzzitivity of the monitored system is a distortion of the market by existing government and political interference. In the monitored sample of variables it concerns mainly the APT variable which is significantly affected by agricultural subsidies at the national and European levels and deflects behaviour of particular economic market agents [10]. To some extent this also applies to the UNT variable which is affected, for example, by a minimum wage, state employment policy, amount of social benefits and a variety of other interventions that unilaterally deflect labour market out of the free market. Relatively free market environment exists only in the area of construction production. Government interventions that have a tendency to grow definitely increase vagueness of the variable behaviour of the monitored system.

The intermediate effects of the crisis, which changed within the three observed years in its character gradually from a financial crisis to an economic crisis, also have an undoubtable effect on the high fuzzitivity of the investigated system. The crisis at the same time slowly spread from individual market subjects to a crisis of public budgets and state debt crisis. This phenomenon, being much stronger in the Eurozone countries than in the Czech Republic, has an imminent influence upon foreign demand, upon which the Czech economy, being small and open, is to a certain extent very dependent. This is especially the case in agricultural production, half-finished products and food (i.e. generally APT), in a smaller extent also the export of construction materials, construction workforce and investment construction units (generally CPT). Foreign influences, however, tend to show also the other way round. It is mostly the large import of agricultural production into the Czech Republic, where also typical and traditional agricultural products of both cattle- and plant production are being imported into the Czech Republic, as well as technical and construction material. This import narrows down the operating space for traditional Czech manufacturers and their supply is limited. The foreign influences have the largest affect upon the variable UNT, as the free movement of labour force is one of the freedoms of the EU free market. The analysed economical areas of APT and CPT are rather less demanding in matters of the labour force qualifications, therefore they are most affected by the tide of foreign labour force. This feature cannot be influenced on the national level, thus it has an imminent influence upon the growth of UNT and is one of the reasons for its high fuzzitivity.

Despite the above mentioned facts, it is still possible to observe dependencies in the monitored variables sample; the dependencies can be demonstrated by this work. In 2009 and 2011, there was a similar trend of CPT and APT seasonal cycles in summer (from June to September) with a demonstrated decrease of UNT while in 2011 this trend was even stronger than in 2009. It is a well-known phenomenon of production growth (in this case, CPT and APT) with a parallel decrease of unemployment during summer, or more precisely, with the rise of unemployment during winter, the so-called seasonal unemployment. In 2010, the system behaved fuzzily with an unproven dependency of CPT and APT on UNT. In 2010 the cycle amplitude of APT was significantly lower than the CPT amplitude, which can be adequately explained by elasticity of demand, i.e. the proportion of the change in the demanded quantity and price. Elasticity in the APT area (together agricultural production and food) is much lower, sometimes almost zero, compared to elasticity of CPT (together private and public construction), where elasticity is high. Therefore, trends zone and seasonal cycles of APT are significantly narrower than the CPT ones and sustainably achieve smaller fluctuations. The same is also fuzzitivity of the relation between the APT and UNT variables, which is significantly lower than between CPT and UNT.

Comparing the trends zones of CPT and APT in relation to UNT in the monitored period in Czech conditions we can demonstrate the existence of the so-called Okun's law. It is an empirical relationship between cyclical movements of GDP (in our case the CPT and APT variables) and UNT. The law says that if a real GDP drops towards a potential one by 2%, the unemployment rate (in our case UNT) will increase by approximately 1%. This relationship applies to the total GDP (not only to the sum of CPT and APT); however, contradictory movement of these variables can be proved by this work. While APT trend zone is growing (CPT relatively

stagnant), the UNT trend zone is decreasing, i.e. when production increases unemployment decreases. This phenomenon can be observed in the variables during the period of 2009-2011, while in the last year of the period the phenomenon shows itself most strongly.

All the variables investigated above have an immediate effect on the fiscal area of the economic policy of the state. Whilst the level of production of the real GDP (parts of which are also APT and CPT) affects the level of public costs tax allocation, the level of UNT affects the level and rate of their later redistribution. However, the decrease in demand for APT and CPT has via its influence over the drop in price level and immediate affect on the monetary area of the economic policy of the state. Economic entities then react to the decrease in the expected inflation by trying to obtain interest-bearing assets by selling other assets. This way, they are trying to lower the losses from holding liquid assets that they got by the continuous inflation. Such purchases of new assets, however, lead to growth in their price and drop in their real pay-off, meaning that even an expected increase in inflation will lead to a lowering of the interest rate. In economic literature, this effect is called the Mundell-Tobin effect.

From the above shown outputs of the time series analysis of UNT, CPT and APT we can find out and especially demonstrate the interdependence of the variables described above, and so in some points even their high fuzzitivity. This is mainly due to the global nature of the economy, protracted economic crisis, time delays and mainly state interventions and political measures which influence free market and the national economy.

Conclusions

In classical statistical regression, we assume that the relationship between dependent variables and independent variables of a model is well-defined and sharp. Although statistical regression has many applications, problems can occur in the situations in which number of observation is inadequate (small data set), difficulties verifying distribution assumptions exists, vagueness in the relationship between input and output variables exist, the ambiguity of events or degree to which they occur or inaccuracy and distortion introduced by linearization is possible.

However, in the real world, it is hampered by the fact that this relationship is more or less non-specific and vague. The suitable theoretical background for abstract formalization of the vague phenomenon of complex systems is the fuzzy set theory. In the paper vague data is defined as specialized fuzzy sets - fuzzy numbers and a fuzzy linear regression model as a fuzzy function with fuzzy numbers as vague parameters. The determination of regression model uncertainty using fuzzy approaches does not require meeting the above presumptions.

To identify the fuzzy coefficients of the model, the genetic algorithm is used. The linear approximation of the vague function together with its possibility area is analytically and graphically expressed. The suitable application is performed in the task of the time series fuzzy regression analysis. The time-trend and seasonal cycles including their possibility areas of selected economical dependences are calculated and expressed, namely time-development of unemployment, agricultural production and construction.

A number of assumptions, concerning the development of the CPT, APT and UNT variables, their seasonality and also relationship between them, were proved by the performed fuzzy regression analysis of the selected variables and also by professional works and empirical observations. In the first (2009) and third (2011) years of observation there was a common and seasonal growth of CPT and APT, while the growth of CPT was lower due to higher elasticity of demand for construction, as well as to the full impact of the economic crisis in this segment of economy. The assumption that during the studied period UNT drops along with CPT and APT growth was also confirmed; that fact was successfully demonstrated by the fuzzy regression analysis.

However, the fuzzy regression analysis of the time series development of CPT, APT and UNT demonstrated a non-standard behaviour of the monitored variables in 2010; from an economic point of view this is a result of a number of causes. Here we talk about the third and deepest crisis year and we can see full influence of the state and a huge impact of globalization on the small and open economy of the CR. The delay, certainly, played its role here; it appeared in the economy during the second studied year (2010). This year, the system of indicators behaved fuzzily and the interdependence of CPT and APT on UNT was never proven by the model; moreover, the model behaved much vaguer, i.e. fuzzily, in relation of CPT to UNT than in relation of APT to UNT. The cause of the phenomenon we can find in low elasticity of demand for agricultural production, or, for example, in

the rising price of agricultural commodities throughout the period. State intervention and transnational influences on the APT and UNT variables are so large that they can be seen as one of the causes of non-standard and fuzzy behaviour of these variables during the year. The effect of efficiency functions vagueness was commented.

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