Forecasting the Trend of Art Market

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Abstract
The paper discusses two different methods to forecasting the global index of Art Market: a Holt-Winters type exponential smoothing method for times series with additive components (time trend and seasonal variation) and a Seasonal Autoregressive Integrated Moving Average (SARIMA) model. Both methods point out that the decline of Art Market started in 2015 will continue in 2018 and 2019, and a slight recovery will be possible by 2020. We also presented a method for combining forecasts.

Keywords: Index of Art Market, forecasts, Holt-Winters method, SARIMA model.

JEL Classification: C22, C53, Z10

Introduction
In Eli Anapur terms, "any guesses regarding the art market trends may seem an impossible, if not an obsolete task" (Anapur 2016). This is because, when it comes to making forecasts, it should be taken into account that the art market is extremely volatile. An authority in the field of art market, explained this thing: "my annual attempt at gazing into my crystal ball to predict what will happen in the coming year in the art market has been comprehensively blown up by a single event in 2017: the astonishing, record-shattering, beyond-predictable price made by Leonardo Da Vinci’s Salvator Mundi" (Adam 2018). On the other hand, the Deloitte Art & Finance Report 2017 predicts that the UHNWI [Ultra High Net Worth Individual – our note] will allocate US$2.706 trillion by 2026 to art and collectibles, as against to US$1.622 trillion allocated in 2016 (Deloitte 2017, 36).

In the same field, of art market, we mention the paper of (Filipiak and Filipowska 2016), that analysed art price databases, the price indices (calculating for the purpose of "measuring financial performance, evaluating diversification of a potential portfolio and describing trends on the market") and evaluated the employment of IT support in art market analysis.

As a methodological approach, we point out the (Jurevičienė and Kostecka 2014) study, that applied ARIMA method to forecast "changes of art prices in 2014-2015 for the aggregate Artmarket Global index and for different art movements (old masters, impressionism and post-impressionism, modern art, contemporary art)". As the structure of the models, by using data from 1998q1 to 2013q2, they have chosen the description ARIMA (5,1,1) for Old Masters Artprice Global Index (Jurevičienė and Kostecka 2014, 78) and an ARIMA(2,1,2) structure for Impressionism and Post-Impressionism Index, for Modern art Index and for Contemporary Art Index (pp. 79-80).

1. Data and Methodology of Forecasting

1.1. Data
We extract the data concerning the Index of Art Market comes from Copyright Artprice.com available at http://imgpublic.artprice.com/pdf/agi.xls. Indexes are calculated based on pricing for auction results: "All the prices [...] indicate auction results – including buyer’s premium – at public sales of Fine Art. [...] Fine Art means paintings, sculptures, drawings, photographs, prints, videos, installations, tapestries, but excludes antiques, anonymous cultural goods and furniture" (according to the methodological note from https://www.artprice.com/artprice-reports/the-art-market-in-2017). The data, calculated in euro, for 1998q1, to 2018q1 are detailed in Annex 1 and depicted in figure 1.
1.2. The forecasting methods

To forecasting the trend of art market we have used two classes of methods. On the one hand we apply a Holt-Winters type exponential smoothing method for times series with additive components (time trend and seasonal variation) and, on the other hand, we apply a Seasonal Autoregressive Integrated Moving Average (SARIMA) models.

According to Holt-Winters exponential smoothing method, the time series $y_t$ can be written as follows (Jula and Jula 2018, 167-198):

$$y_t = L_t + T_t + S_t + \epsilon_t,$$

where $\epsilon_t$ is the disturbances variable
and $L_t$ smoothing mean series level at the $t$ moment;
$T_t$ trend at the $t$ moment;
$S_t$ additive seasonal coefficient the $t$ moment.

The three components are calculated by the following recursive relationships:

$$L_t = \alpha(y_t - S_{t-s}) + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

$$S_t = \gamma(y_t - L_t) + (1 - \gamma)S_{t-s},$$

where $0 \leq \alpha, \beta, \gamma \leq 1$ are the damping factors and $s$ is the seasonal frequency ($s = 4$ for quarterly data).

Forecasts are computed through the following relationship:
Seasonal Autoregressive Integrated Moving Average (SARIMA) models are denoted SARIMA(p,d,q)(P,D,Q)s, where "p is the order (number of time lags) of the autoregressive model, d is the degree of differencing [i.e. the necessary number of differentiations to ensure the stationarity of the series, own note], and q is the order of the moving-average model. Similarly, P, D and Q refer to the autoregressive, differencing, and moving average terms for the seasonal part of the SARIMA model, and s refers to the number of periods in each season" (Jula and Jula 2018). In this model, all the parameters (both p, d, q, and P, D, Q, and s) are non-negative integers.

In the following models we adopted the relationships and notations described in (Jula and Jula 2018). The ARMA(p,q) models can be write as follow:

\[ y_t - \mu - \phi_1 y_{t-1} - \phi_2 y_{t-2} - \ldots - \phi_p y_{t-p} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q}. \]

where \( \phi \) are the coefficients from autoregressive part of the process, \( \theta \) are the coefficients from the moving average part, \( \mu \) is the mean of the time series and \( \varepsilon_t \) is an error term, usually, a random (normal) i.i.d. variable (i.e. white noise).

By using the lag operator, defined as \( L y_t = y_{t-1} \), ARMA model is given by:

\[ \Phi(L)y_t = \mu + \Theta(L)\varepsilon_t, \]

where \( \Phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p \) is the polynomial for the autoregressive part and \( \Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_q L^q \) stands for the polynomial aimed at the moving average part.

Through defining the differentiation operator \( \Delta = 1 - L \), i.e. \( \Delta y_t = y_t - y_{t-1} \), ARIMA(p,d,q) process can be written as follow:

\[ \Phi(L)(1 - L)^d y_t = \phi_0 + \Theta(L)\varepsilon_t, \]

and SARIMA(p,d,q)(P,D,Q)s model is given by

\[ \Phi(L)\phi(L)^s(1 - L)^d(1 - L^s)^D(y_t - \mu) = \Theta(L)\Theta(L^s)\varepsilon_t. \]

The seasonal parts of the model are constructed for autoregressive seasonal part by the polynomial \( \phi(L^s) = 1 - \phi_1 L^s - \phi_2 L^{2s} - \ldots - \phi_p L^{ps} \) and \( \Theta(L^s) = 1 - \theta_1 L^s - \theta_2 L^{2s} - \ldots - \theta_q L^{qs} \), for moving average seasonal part. We selected the periodicity of series at \( s = 4 \) (quarters) and used SARIMA(p,d,q)(P,Q)s models to forecast the trend of art market, over the 2018 - 2020 years.

2. Outcomes of Forecasting the Global Index of Art Market

2.1. Holt-Winters Exponential Smoothing Method

We have applied a Holt-Winters Exponential Smoothing Method with additive components (time trend and seasonal variation). The outcomes from the model with multiplicative components do not differ significantly from the additive model. The detailed EViews-10 solution for the series Global Index of Art Market is the following:

<table>
<thead>
<tr>
<th>Table 3. Holt-Winters Exponential Smoothing Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample: 1998Q1 2018Q1</td>
</tr>
<tr>
<td>Included observations: 81</td>
</tr>
<tr>
<td>Method: Holt-Winters Additive Seasonal</td>
</tr>
</tbody>
</table>

\[ F_{t+h} = L_t + hT_t + S_{t+h+s}. \]
Given the fact that the values estimated through the Holt-Winters exponential model are \( \alpha = 0.66, \beta = 0 \) and \( \gamma = 1 \), the projected model (for periodicity s = 4) is

\[
L_t = 0.66(y_t - S_{t-s}) + 0.44(L_{t-1} + T_{t-1}),
\]

with \( L_{2018q1} = 136.2456 \)

\[
T_t = T_{t-1} = 0.532895
\]

And \( S_t = y_t - L_t = \begin{cases} -5.112847, & \text{for } q = 1 \\ 4.556749, & \text{for } q = 2 \\ -10.53401, & \text{for } q = 3 \\ 11.09011, & \text{for } q = 4 \end{cases} \)

Source: Author’s calculation in EViews, based on Artprice data, available at http://imgpublic.artprice.com/pdf/agi.xls
The forecasts are computed through the following relationship $F_{t+h} = L_t + hT_t + S_{t+h-4}$, for $h = 1$ to $11$, i.e. the time from 2018q2 ($h = 1$) to 2020q4 ($h = 11$). The series \textit{Global Index of Art Market (GIE)} and the forecasting obtained by Holt-Winters exponential smoothing model for times series with additive components are described in figure 2. The Holt-Winters Exponential Smoothing model estimates that, for the global index of the art market, the fall registered between 2015-2017 will be stopped only in 2019 and even it will register a slight return over the next years. The average 2018 projected index is 138.6, compared to 148.9 in 2017, while for 2019 is estimated at and 139.2 and 141.3 points for 2020.

2.2. SARIMA models

For the alternative forecasting, we have applied a SARIMA type model. Concretely, in SARIMA(p,d,q)(P,Q)4 we have fixed the largest number of differences to $d = 2$, the maximum order of autoregressive terms to $p = 12$ (i.e. twelve quarters = 3 years), and for moving average part, the choice for the maximum order was, also, $q = 12$ (quarters). The largest values for the seasonal components was restricted at 2 seasons, both for autoregressive, and for the moving average part: $P = 2s$, and $Q = 2s$ where the periodicity is $s = 4$ quarters). Too, the series concerning the global index of the art market (GIEAM) has been studied both in the level, and through the logarithmic transformation. With these chosen values, the possible number of SARIMA models was 1521. In order to calculate the model, we used the EViews-10 software package. Of all these 1521 possible combinations, the model that minimizes the Schwarz Information Criterion (SIC) was found to be SARIMA(1,1,1)(1,0)4, applied on ln(GIEAM). The differencing selection order was performed through KPSS test (with 10% level of significance). The estimators of the model are the following (table 2):

\begin{table}[h]
\centering
\caption{SARIMA model for Global Index of Art Market}
\begin{tabular}{|c|c|}
\hline
Selected dependent variable: & dlog(GIAM) \\
Number of estimated ARMA & models: 1521 \\
Selected model: & (1,1,1)(1,0) \\
\hline
\end{tabular}
\end{table}
Sample: 1998Q2 2018Q1 (80 observations)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.003346</td>
<td>0.009452</td>
<td>0.354008</td>
<td>0.7243</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.978432</td>
<td>0.037069</td>
<td>-26.39463</td>
<td>0.0000</td>
</tr>
<tr>
<td>SAR(4)</td>
<td>0.260329</td>
<td>0.094490</td>
<td>2.755098</td>
<td>0.0074</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.813773</td>
<td>0.077796</td>
<td>10.46030</td>
<td>0.0000</td>
</tr>
<tr>
<td>SIGMASQ</td>
<td>0.004183</td>
<td>0.000648</td>
<td>6.456726</td>
<td>0.0000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.578560</td>
<td></td>
<td></td>
<td>-2.493386</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>2.005832</td>
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<td>-2.344509</td>
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<tr>
<td>F-statistic</td>
<td>25.74031</td>
<td></td>
<td></td>
<td>-2.433697</td>
</tr>
</tbody>
</table>

Source: Author’s calculation in EViews-10, based on Artprice data, available at http://imgpublic.artprice.com/pdf/agi.xls

The detailed calculations are presented in Annex 2. The series Global Index of Art Market and the forecasting obtained by SARIMA model are depicted in figure 3.

Figure 15. Actual data and forecast (SARIMA model) for Global Index of Art Market

According to the SARIMA(1,1,1)(1,0)4 model, the Global Index of Art Market will continue the fall that started in 2015, until 2019, then a smooth recovery process will begin. The average 2018 projected index is 137.1, and it is 135.9 for 2018, compared to 147 in 2017, while for 2020 is estimated a return to 137 points.
A comparison between the two forecasting methods (Hodrick-Prescott and SARIMA) is depicted in figure 4.

![Figure 16. Forecasting of Global Index of Art Market – Holt-Winters and SARIMA methods](image)

**Source:** Author’ calculation in EViews-10, based on Artprice data, available at http://imgpublic.artprice.com/pdf/agi.xls

### 2.3. Combining Forecasts

**Forecast averaging** combines a variety of disponible forecasts, for each out-of-sample observation, into a single result, by calculating a weighted average of multiple forecasts. There are several ways to combine forecasts (Steel 2017). We used the classical (Stock and Watson 2004) Mean Square Error (MSE) weighting method. For each forecast \( i \) is computed a weight as:

\[
 w_i = \frac{1}{\sum_{j=1}^{k} \frac{1}{\text{MSE}_j^k}}
\]

where \( \text{MSE}_i \) is the mean square error of forecast \( i \), who is computes over some in-sample period, while \( k \) is a power. Usually, is taken \( k = 1 \), and we assume this value in our calculations.

The in-sample period taken into consideration is 2000q1-2018q1. Over this period, the sum square error (SSE) for the two forecasts (Holt-Winters and SARIMA) does not differ greatly. They have the following values:
SS_{HoltWinters} = 6420.51 and SS_{SARIMA} = 6593.86. As a result, the two weights do not significantly differ, more exactly they are: \( w_{HoltWinters} = 0.5067 \) and \( w_{SARIMA} = 0.4933 \). The results are depicted in the figure 5.

Figure 17. Combining forecasts

The average Global Index of Art Market is projected at 139.2 points in 2020 (with seasonal variations), increasing compared to 2018 and 2019, but still below the average 2017 level (147 points).

Conclusions

To forecast the Global Index of Art Market we have used a Holt-Winters type exponential smoothing method for times series with additive components (time trend and seasonal variation) and a Seasonal Autoregressive Integrated Moving Average (SARIMA) model. By running 1521 models type SARIMA\((p,d,q)(P,Q)s\) (for \( p \) and \( q \) between 0 and 12, \( d \) between 0 and 2, \( P \) and \( Q \) between 0 and 2s), we selected SARIMA\((1,1,1)(1,0)s\) (this is the model that minimizes the Schwarz Information Criterion (SIC)).

Both methods (Holt-Winters) point out that the decline of Art Market started in 2015 will continue in 2018 and 2019, and a slight recovery will be possible by 2020. Concretely, the Holt-Winters Exponential Smoothing model estimated that the fall registered between 2015-2017 to Art Market will be stopped only in 2019 and even it will register a slight return over the next years. The average 2018 projected index is 138.6, compared to 148.9 in 2017, while for 2019 is estimated at and 139.2 and 141.3 points for 2020. Similarly, according to the SARIMA\((1,1,1)(1,0)s\) model, the Global Index of Art Market will continue the fall that started in 2015, until 2019, then a smooth recovery process will begin. The average 2018 projected index is 137.1, and it is 135.9 for 2018, compared to 147 in 2017, while for 2020 is estimated a return to 137 points.
We also presented a method for combining forecasts. As averaging forecasts methodology, we used a (Stock and Watson 2004) Mean Square Error (MSE) weighting method. The average Global Index of Art Market is projected at 139.2 points in 2020 (with seasonal variations), increasing compared to 2018 and 2019, but still below the average 2017 level (147 points).

Annexes

Annex 1. Global Index of Art Market

<table>
<thead>
<tr>
<th></th>
<th>Global Index of Art Market</th>
<th>Global Index of Art Market</th>
<th>Global Index of Art Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998q1</td>
<td>100</td>
<td>2005q1</td>
<td>137</td>
</tr>
<tr>
<td>1998q2</td>
<td>112</td>
<td>2005q2</td>
<td>145</td>
</tr>
<tr>
<td>1998q3</td>
<td>109</td>
<td>2005q3</td>
<td>147</td>
</tr>
<tr>
<td>1998q4</td>
<td>105</td>
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<td>145</td>
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<tr>
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<td>152</td>
</tr>
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<td>157</td>
</tr>
<tr>
<td>1999q3</td>
<td>113</td>
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<td>1999q4</td>
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<td>2000q4</td>
<td>137</td>
<td>2007q4</td>
<td>142</td>
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<tr>
<td>2001q1</td>
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<td>167</td>
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<td>140</td>
<td>2008q2</td>
<td>150</td>
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<tr>
<td>2001q3</td>
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<td>2008q3</td>
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<tr>
<td>2001q4</td>
<td>131</td>
<td>2008q4</td>
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<tr>
<td>2004q4</td>
<td>136</td>
<td>2011q4</td>
<td>153</td>
</tr>
</tbody>
</table>

Annex 2. Automatic SARIMA model selection to forecasting the Global Index of Art Market (GIEAM)

Selected dependent variable: dlog(GIAMI)

Number of estimated ARMA models: **1521**.

Selected model: (1,1,1)(1,0)

Method: ARMA Maximum Likelihood (BFGS)

Sample: 1998Q2 2018Q1

Included observations: 80

Convergence achieved after 35 iterations

Coefficient covariance computed using outer product of gradients

<table>
<thead>
<tr>
<th>Variable</th>
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<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared     | 0.578560    | Mean dependent var | 0.004558 |

Adjusted R-squared | 0.556083 | S.D. dependent var | 0.100253 |

S.E. of regression | 0.066795 | Akaike info criterion | -2.493386 |

Sum squared resid | 0.334621 | Schwarz criterion | -2.344509 |

Log likelihood   | 104.7354   | Hannan-Quinn criter. | -2.433697 |

F-statistic      | 25.74031   | Durbin-Watson stat | 2.005832 |

Prob(F-statistic) | 0.000000  |                  |        |

| Inverted AR Roots | .71 | .00-.71i | -.00+.71i | -.71 |

| Inverted MA Roots | -.81 |        |          |      |

Actual and Forecast Global Index of Art Market

Source: see Annex 1

Forecast Comparison Graph

Source: see Annex 1
## Detailed Forecast Comparison Graph

Source: see Annex 1

### References


