Structural Breaks in Potential GDP for Romanian Economy

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Abstract
In the paper we have calculated the potential gross domestic product for Romanian economy, by using both the trend functions (deterministic and stochastic) and the filters (Hodrick-Prescott filter and band-pass filters). We found that the log-quadratic deterministic trend with structural breaks method offer the best outcomes. Bai-Perron tests has selected 1999q1 and 2009q1 as moments of structural breaks. According this technique, the Romanian potential GDP was over 6% during the 2004-2008 period and returns to around 6% in the years 2016-2017. With a few exceptions (recorded before 2010), output gap was above -0.5 and below +0.5% of GDP. This mean that the evolution of economic growth in Romania (especially after 2010) do not has induced inflationary pressures, through supply-side.

Keywords: potential gross domestic product, output gap, trend functions, structural breaks filters

JEL Classification: C22, C24, O11

Introduction
In Gavin (2012) words, potential gross domestic product (GDP) is "a theoretical concept that means different things to different people" (Gavin 2012, 1). Hence, in the economic literature there are several definitions of potential gross domestic product (GDP).

In the OECD’s Economic Outlook publication, potential GDP is defined "as the level of output that an economy can produce at a constant inflation rate […] Potential output depends on the capital stock, the potential labour force (which depends on demographic factors and on participation rates), the non-accelerating inflation rate of unemployment (NAIRU), and the level of labour efficiency" (OECD 2014). European Union adopts a similar definition: "Potential output is a concept used to measure the highest level of production that an economy can reach without generating inflationary pressures" (European Parliament 2015).

In Gavin opinion, "[monetary policymakers] estimate potential GDP by constructing measures of the trend in actual GDP that smooth out business cycle fluctuations" (Gavin 2012, 1). Therefore, in Romania, the national monetary policymakers, namely the National Bank of Romania (NBR) keeps the following definition: "The equilibrium level of GDP at which the ratio between the capital stock and the amount of labour employed generates no inflationary pressures is called the potential GDP." (National Bank of Romania August 2005, 41). Recently, the National Commission for Prognosis (NCP) keep hold of a more complex definition: "Potential GDP stands for a stable level of gross domestic product corresponding to the aggregate supply curve over the medium and long term, so that the transient effects of macroeconomic distortions disappear. Potential GDP explains that growth in relative equilibrium conditions, i.e. without pressure on inflation and employment" (National Commission for Prognosis 2018, 4).

Whatever the method of calculating potential GDP, output gap is calculated as the difference between a real (actual) level and the potential level of gross domestic product:

\[ \text{GDP Gap} = \text{Real (actual) GDP} - \text{Potential GDP}. \]

Potential gross domestic product and output gap are some of the most widely used variables by policymakers in building monetary and fiscal macroeconomic policies. In European Union, "the surveillance of fiscal policies of EU Member States makes extensive use of estimates of the potential output and related concepts, including output gap and structural budget balance" and "the role of output gap estimates in economic analysis and fiscal policy is twofold: to quantify the nature of the economic cycle and identify the actual economic situation within the cycle, and to suggest counter-cyclical economic policies aimed at influencing the length and the effects of the cycle itself" (European Parliament 2015).
Table 1. Annual fiscal adjustments towards the medium-term objective (MTO) under the preventive arm of the Stability and Growth Pact (SGP)

<table>
<thead>
<tr>
<th>Economic cycle</th>
<th>Condition</th>
<th>Required annual fiscal adjustment (in percentage points of GDP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Debt below 60 % and no sustainability risk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>growth &lt; potential</td>
</tr>
<tr>
<td>Exceptionally bad</td>
<td>real growth &lt; 0 or output gap &lt; -4</td>
<td>no adjustment needed</td>
</tr>
<tr>
<td>Very bad times</td>
<td>-4 ≤ output gap &lt; -3</td>
<td>0</td>
</tr>
<tr>
<td>Bad times</td>
<td>-3 ≤ output gap &lt; -1.5</td>
<td>0</td>
</tr>
<tr>
<td>Normal times</td>
<td>-1.5 ≤ output gap &lt; 1.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Good times</td>
<td>output gap ≥ 1.5 %</td>
<td>0.5</td>
</tr>
</tbody>
</table>


1. Basic methods for potential gross domestic product and output gap calculation

The basic methods for potential gross domestic product (and thereby, output gap) calculation can be laid out into three broad classes: methods derived from economic theory (structural methods); a-theoretical methods (that determine the trend by using statistical techniques – trend function and/or filters); and mixed methods (that combine statistical procedures with elements based on relations brought from economic theory).

1.1. Methods derived from economic theory

These methods start from the fact that potential output is a measure of production capacity of the economy, therefore any estimation of potential output must be based on explicit dependence of inputs. This relationship between inputs (usually, capital, labour, and total factors productivity) and outputs is expressed through a production function, most often by the Cobb-Douglas type production function. This type of calculation allows to draw out the contribution of each factor to (real or potential) economic growth (i.e. bringing to light the sources of economic growth).

As disadvantages, we notice that methods derived from economic theory are based on the acceptance of strongly restrictive assumptions, which may not be consistent with the data. More exactly, some values are required for model parameters - for example, the factors elasticity in the production function (take usually as a steady weight of labour costs in production value). Moreover, as a rule, the models derived from economic theory use deterministic trends for cyclical adjustment of potential output. But, there are empirical studies on business cycles that shows that variable trends are more proper than deterministic ones to most macroeconomic data series. Additionally, models based on economic theory typically use estimations of some thresholds (e.g. non-accelerating inflation rate of unemployment – NAIRU). And a such estimates are uncertain. Or, the models use various techniques (accept assumptions) to evaluate the expected values of some variables (e.g., expected inflation). And, by that, uncertainty about the potential GDP determination is transferred to the NAIRU calculation.

1.2. A-theoretical methods

Broadly speaking, these methods estimate the trend of output (e.g. GDP) by using statistical techniques – trend function and/or filters. As advantages, we bring up the fact that, technically, they are simple (easy) to apply, are transparent and easy to understand. Furthermore, the statistical techniques do not depend by the context in which they are applied (e.g. the nature, or the significances of data series), or by the constraints imposed by a certain economic theory.
As disadvantages, we point out that they are not explicitly rooted in economic theory. As United States Congressional Budget Office emphasizes: "they provide a measure of trend output but not potential output" (Congressional Budget Office 2004, March, 2). Likewise, technically, these methods arise the risk of generating false cycles.

These methods try to estimate the trend for the whole-time series. To achieve this goal, a-theoretical methods use trend functions (deterministic or stochastic trend) or apply different filters (e.g. Hodrick-Prescott, Band-Pass Filters).

A summary of different methods used for estimating potential GDP can be found in (Congressional Budget Office 2004, March).

2. Potential gross domestic product in Romania

2.1. Economic literature on Romanian potential GDP

A 2007 study, conducted at the National Bank of Romania (Gălățescu, Rădulescu and Copaciu 2007) has set as objective "to estimate the growth rates of potential GDP for Romania". To this end, the authors "implement several univariate and multivariate methods for the measurement of potential GDP growth: production function, filters with unobservable components, structural vector autoregressions" (Gălățescu, Rădulescu and Copaciu 2007, 3). They found that "the estimates arrived at with different methods and specifications offer similar conclusions" (Gălățescu, Rădulescu and Copaciu 2007, 16). The authors’ main findings are synthesized in the following statements: "The results indicate an increasing annual potential GDP growth rate, from an average of 3-4 percent in the 2000-2002 period to values of around 6 percent in recent periods [2003q1-2006q2, our note]. Thus, Romania’s potential GDP has grown at rates significantly above those registered by new EU Central and Eastern European member states and South European states in their periods of high growth; the magnitude of Romania’s growth rates is comparable to that of growth rates in the Baltic countries, being exceeded only by the size of growth rates estimated for Ireland and Latvia" (Gălățescu, Rădulescu and Copaciu 2007, 16).

By using both a HP filter and a Cobb-Douglas production function (Andrei and Păun 2011, 19) estimated the growth of potential GDP for Romania between -0.36% and 2% during the 1994-2000 period and an increase to 5%-6.7%, over 2004-2008. In a later article, (Andrei and Păun 2015) used more filters (Hodrick-Prescott, Baxter-King, Christiano-Fritzgerald) and the same Cobb-Douglas production function. The authors found that the Christiano-Fritzgerald band-pass filters performed better for the end to extract Romanian potential GDP over the period 2000-2012.

(Altăr, Necula and Bobeică 2010) estimated potential GDP for the Romanian economy by using a production function structural method, combined with several filtering methods. Authors found "for the period 2001-2008 an average annual growth rate of the potential output equal to 5.8%, but on a descending slope, due to the adverse developments in the macroeconomic context" (Altăr, Necula and Bobeică 2010, 5).


<table>
<thead>
<tr>
<th>Year</th>
<th>2016</th>
<th>2017</th>
<th>2018</th>
<th>2019</th>
<th>2020</th>
<th>2021</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential GDP (% change on previous year)</td>
<td>4.3</td>
<td>4.7</td>
<td>5.2</td>
<td>5.4</td>
<td>5.3</td>
<td>4.9</td>
</tr>
</tbody>
</table>


2.2. Estimating the potential GDP for Romanian economy

In figure 1, we depict the Romanian Gross Domestic Product, where the data are for the period between 1995 and 2017. On these data we estimate the potential GDP.
Deterministic Trend

Log-linear Deterministic Trend

For the time series data depicted in figure 1 (GDP means Gross domestic product at market prices, quarterly data, chain linked volumes (2010), million units of national currency, seasonally and calendar adjusted data), we first estimate the potential GDP through the log linear deterministic trend (figure 2). The results are the following:

Model: \( \ln(GDP_t) = a_0 + a_1t + e_t \),

Estimated model: \( \ln(GDP_t) = 11.30363 + 0.00799 \cdot t, \ R^2 = 0.9194 \)

\( (0.01314) \quad (0.00025) \)

(in parenthesis, under estimators: standard deviation)
Figure 2. Potential GDP estimates using log-linear deterministic trend

Source: authors' calculations in EViews-10, based on the Eurostat (table namq_10_gdp) data (http://ec.europa.eu/eurostat/data/database)

Log-Quadratic Deterministic Trend with Structural Breaks

The data depicts in figure 2 suggest a more complex dynamic of real and potential GDP. Therefore, we built a deterministic log-quadratic trend model with structural breaks (figure 3). The model is:

\[ \ln(GDP_t) = a_0 + a_1t + a_2t^2 + e_t \]

(Method: Least Squares with Breaks: Break type: Bai-Perron tests of L+1 vs. L sequentially determined breaks, Breaks: 1999Q1, 2009Q1). EViews-10 solutions:

1995q1 – 1998q4:
\[
\ln\left(\text{GDP}_t\right) = 11.39502 + 0.007366 \cdot t - 0.000730 \cdot t^2
\]

1999q1 – 2008q4:
\[
\ln\left(\text{GDP}_t\right) = 11.23071 + 0.004148 \cdot t + 0.000149 \cdot t^2
\]

2009q1 – 2017q4:
\[
\ln\left(\text{GDP}_t\right) = 12.83203 - 0.033954 \cdot t + 0.000280 \cdot t^2
\]

R² = 0.9971, dw = 1.36

By combining the previous relationships, we obtain the following model:

\[
\ln(GDP_t) = @before("1999q1") \cdot (11.39502 + 0.007366 \cdot t - 0.00073 \cdot t^2) + \\
+ @during("1999q1 2008q4") \cdot (11.23071 + 0.004148 \cdot t + 0.000149 \cdot t^2) + \\
+ @after("2009q1") \cdot (12.83203 - 0.033954 \cdot t + 0.000280 \cdot t^2)
\]
Bai-Perron tests has selected 1999q1 and 2009q1 as moments of structural breaks. Both moments are explicable, given the economic developments in Romania. The first, 1999q1, marks the resumption of economic growth after the strong depression registered in the years 1997-1998, and the second moment, 2009q1, is related to the economic and financial crisis registered at the end of the first decade of this century.

**Stochastic Trend**

Beveridge-Nelson (1981) proposes a method of decomposing a non-stationary series into a permanent component (trend) and a transitory (cyclic) component, by using an ARIMA(p, 1, q) model. The procedure allows both components to be stochastic. The trend ($\tau_t$ – the permanent component) is produced by a series of non-predictable shocks (innovations) and is modelled as a random walk with drift.

$$y_t = y_{t-1} + e_t,$$

where $e_t$ is an ARMA(p,q) process.
Beveridge and Nelson decompose the time series \( y_t = y_{t-1} + e_t \) [where \( e_t \) is an ARMA(p,q) process] into two parts:

\[
y_t = \tau_t + \xi_t
\]

where the permanent component is

\[
\tau_t = \tau_{t-1} + \varepsilon_t.
\]

(\( \varepsilon_t \) is white noise). Also, the cyclic (transitory) component (\( \xi_t \)) is a *stationary process with zero mean*.

To apply a such model, first we test if real GDP are I(1), and if the shocks may be described as ARMA process.

Indeed GDP is nonstationary I(1) process, even if we admit the structural breaks in trend (the tests are detailed in Annex A1). But, \( \xi_t = d(GDP_t) \) is not an ARMA process. According to the BDS test and the Variance Ratio Test, \( d(GDP) \) is a series with i.i.d. values (the tests are detailed in Annex A2). Nevertheless, for comparison, we apply the Beveridge-Nelson decomposing on GDP. The results are depicted in figure 4.

### Filters

**Hodrick-Prescott Filter**

The Hodrick-Prescott Filter (1981, 1997) is one of the most popular and used methods for extracting a trend from a time series. Let \( X_t \) a time series which should be decomposed into two components: a trend (\( TR_t \)) and a cyclic component (\( C_t \)):

\[
X_t = TR_t + C_t.
\]

The Hodrick-Prescott filter extracts the trend by minimizing the following expression:
\[ \sum_{t=1}^{T} (x_t - TR_t)^2 + \lambda \sum_{t=2}^{T-1} \left( (TR_{t+2} - TR_t)^2 - (TR_t - TR_{t-1})^2 \right) \]

The first term measures the degree of deviation from the trend, and the second term is a measure of smoothing the series. Because there is a conflict between adjustment and smoothing, a compromise parameter is used, \( \lambda \). If \( \lambda = 0 \) then the trend becomes equal to the initial series, and if \( \lambda \) goes to infinity, then HP approximates a linear trend. In the literature, \( \lambda = 1600 \) is the value suggested for the quarterly data. For \( \lambda = 1600 \) (GDP is calculated as quarterly data), the HP trend and HP-cycle are depicted in figure 5.

**Band-Pass filters**

**Band-Pass filters** are used to extract the cyclical component of a time series by specifying a range of the cycle (periodicities). For annual frequency data, for example, a filter \((2; 8)\) is usually used, i.e. fluctuations lasting at least 2 years and maximum 8 are considered cyclical. For quarterly Frequency Data, a filter \((6; 32)\) is usually used.

**Symmetric filters**

In symmetric filter the lead and lag length are fixed. The result of applying the Baxter-King symmetric filter (1999) to the GDP series, with cycle low period = 6, and high = 32, is shown in figure 6.

**Figure 5. Potential GDP estimates (trend) and output gap (cycle) using Hodrick-Prescott Filter**

Source: authors' calculations in EViews-10, based on the Eurostat (table namq_10_gdp) data (http://ec.europa.eu/eurostat/data/database)
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GDP Non-cyclical Cycle
The BK filter do not calculate the values for the first 12 and last 12 terms of time series. And this creates difficulties in forecasting the potential GDP.

**Asymmetric filters**

For asymmetric filters the weights on the leads may differ from the and weights on the lags. The result of applying the Christiano-Fitzgerald (2003) asymmetric filter to the GDP series, with I(1) assumption, without detrending method, is shown in figure 7.
3. Comparison of potential GDP estimation methods

Figure 8 depicts the potential GDP estimates for Romanian economy using the six econometric methods: deterministic trend (log-linear and log-quadratic with structural breaks), stochastic trend (Beveridge-Nelson decomposing), Hodrick-Prescott Filter, Band-Pass filters (symmetric and asymmetric). Except for the log-linear trend, the paths followed by the other five estimates are similar.

For comparison, we calculate the sum of squares of differences between real GDP and the potential GDP estimates. The values in figure 9 are compared with those obtained by log-quadratic trend with structural breaks. We do not show the log-linear estimates, because the value is 31.8, much higher than any of the other values.

The most powerful model is the one that is calculated as log-quadratic trend with structural breaks. The detailed shapes are shown in figure 10.
Figure 8. Potential GDP estimates using econometric methods

Source: authors' calculations in EViews-10 based on the Eurostat (table namq_10_gdp) data (http://ec.europa.eu/eurostat/data/database)

Figure 9. Performances of the econometric models in potential GDP estimates

Differences between GDP and Trends
(Log-quadratic Trend with Structural Breaks = 1)

<table>
<thead>
<tr>
<th>Model</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-quadratic Trend (with structural...</td>
<td>1.0</td>
</tr>
<tr>
<td>Christiano-Fitzgerald Trend</td>
<td>3.6</td>
</tr>
<tr>
<td>Beveridge-Nelson Trend</td>
<td>4.5</td>
</tr>
<tr>
<td>Baxter-King Trend</td>
<td>4.8</td>
</tr>
<tr>
<td>Hodrick-Prescott Trend</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Source: authors' calculations in EViews-10 based on the Eurostat (table namq_10_gdp) data (http://ec.europa.eu/eurostat/data/database)
Figure 10. Annual changes in real and potential GDP for Romanian economy

Source: authors' calculations in EViews-10, based on econometric model.

Legend: GDP: Romanian Gross Domestic Product at market prices, quarterly data, chain linked volumes (2010), million ron, seasonally and calendar adjusted data, year % changes

Potential GDP: deterministic log-quadratic trend with structural breaks.
The output gaps calculate from Hodrick-Prescott Filter, Band-Pass Baxter-King symmetric and Christiano-Fitzgerald (2003) asymmetric filter are similar (see figure 10).

**Figure 11. Output gap estimates with Hodrick-Prescott, Band-Pass Baxter-King symmetric and Christiano-Fitzgerald asymmetric filters**

![Graph showing output gap estimates with various filters](image)

Source: authors' calculations in EViews-10 based on the Eurostat (table namq_10_gdp) data (http://ec.europa.eu/eurostat/data/database)

**Figure 12. Output gap – potential GDP estimated as deterministic log-quadratic trend with structural breaks**

![Graph showing output gap with structural breaks](image)

Source: authors' calculations in EViews-10 based on the Eurostat (table namq_10_gdp) data (http://ec.europa.eu/eurostat/data/database)

After 2011, the output gaps calculated through log-quadratic deterministic trend with structural breaks did not exceed ± 0.5 percentage points of real annual GDP (see figure 12). This mean that the evolution of economic growth in Romania (especially after 2010) do not has induced inflationary pressures, through supply-side. However, such pressures existed with positive sign between 1996-2000 and negative sign towards the end of the crisis (2010).
Conclusions

We have calculated the potential gross domestic product for Romanian economy by using a several econometric methods. On the one hand, we used trend functions and filters, on the other hand. As trend functions we calculated deterministic trend (log-linear and log-quadratic with structural breaks) and stochastic trend (Beveridge-Nelson decomposing procedure). As for the filters, we have applied both the Hodrick-Prescott and the band-pass filters (Baxter-King fixed length symmetric filter and Christiano-Fitzgerald full sample asymmetric filter with random walk assumption, without supplementary detrending). We found that the log-quadratic deterministic trend with structural breaks method offer the best outcomes. Bai-Perron test has selected 1999q1 and 2009q1 as moments of structural breaks. Both moments are explicable, given the economic developments in Romania. The first, 1999q1, marks the resumption of economic growth after the strong depression registered in the years 1997-1998, and the second moment, 2009q1, is related to the economic and financial crisis registered at the end of the first decade of this century. According this technique, the Romanian potential GDP was over 4-5% during the 2000-2003, over 6-7% in 2004-2007 period and even 8.1% in 2008. Potential GDP drops dramatically during the crisis (2009-2010) and has risen steadily after 2010, so that it returns to around 6% in the years 2016-2017. With a few exceptions (recorded between 1996-2000, in 2004 and during the economic crisis at the end of the first decade of the current century), output gap was above -0.5 and below +0.5% of GDP. This mean that the evolution of economic growth in Romania (especially after 2010) do not has induced inflationary pressures, through supply-side.

References

Annexes

A1. Testing the nature of GDP series

We tested the hypothesis that the GDP series is stationary with structural breaks. For this purpose, we applied in EViews-10 the Perron Breakpoint Unit Root Test. The outcomes are presented in the following table:

**Table A1: Unit Root with Break Test on GDP**

Null Hypothesis: GDP has a unit root  
Trend Specification: Trend and intercept  
Break Specification: Trend and intercept  
Break Date: 2011Q3  
Break Selection: Minimize Dickey-Fuller t-statistic  
Lag Length: 6 (Automatic - based on Akaike information criterion, maxlag=11)

<table>
<thead>
<tr>
<th>t-Statistic</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-3.475386</td>
</tr>
<tr>
<td>Test critical values:</td>
<td></td>
</tr>
<tr>
<td>1% level</td>
<td>-5.719131</td>
</tr>
<tr>
<td>5% level</td>
<td>-5.175710</td>
</tr>
<tr>
<td>10% level</td>
<td>-4.893950</td>
</tr>
</tbody>
</table>

Source: EViews estimates on Romanian GDP data

The test does not reject the hypothesis of unit root with break (the probability associated to null hypothesis is 0.8248).

Then we tested the hypothesis that the GDP series is stationary by applying the Augmented Dickey-Fuller test, Phillips-Perron test and Ng-Perron optimal tests. We depicted in the following tables only the EViews-10 outcomes for Ng-Perron tests:

**Table A2. Ng-Perron tests for GDP series**

Null Hypothesis: GDP has a unit root  
Exogenous: Constant
Null Hypothesis: GDP has a unit root
Ng-Perron test statistics 2.28199 2.54339 1.11455 107.048

Null Hypothesis: D(GDP) has a unit root
Ng-Perron test statistics -41.5278 -4.55550 0.10970 0.59342
Asymptotic critical values*: 1% -13.8000 -2.58000 0.17400 1.78000
5% -8.10000 -1.98000 0.23300 3.17000
10% -5.70000 -1.62000 0.27500 4.45000

The tests do not reject the hypothesis of unit root for GDP, and do not confirm the unit root for d(GDP). Accordingly, we consider GDP as non-stationary series, more exactly, GDP is an I(1) series.

Annex A2: ARIMA structure of d(GDP) series

We have tested if d(GDP) has or not an ARMA type structure. First, we applied an automatic ARIMA selection for d(GDP)

Table A.3. Automatic ARIMA selection for d(GDP)

<table>
<thead>
<tr>
<th>AR / MA</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18.14196</td>
<td>18.10868</td>
<td>18.11418</td>
<td>18.13155</td>
<td>18.14385</td>
</tr>
<tr>
<td>1</td>
<td>18.09589</td>
<td>18.11815</td>
<td>18.13681</td>
<td>18.15714</td>
<td>18.15251</td>
</tr>
<tr>
<td>2</td>
<td>18.11811</td>
<td>18.12413</td>
<td>18.14232</td>
<td>18.15866</td>
<td>18.16834</td>
</tr>
<tr>
<td>3</td>
<td>18.14022</td>
<td>18.14361</td>
<td>18.16798</td>
<td>18.15523</td>
<td>18.16988</td>
</tr>
<tr>
<td>4</td>
<td>18.12996</td>
<td>18.15206</td>
<td>18.17286</td>
<td>18.17977</td>
<td>18.20113</td>
</tr>
</tbody>
</table>

Then, we estimated the model in the selected structure specification: AR(1,0). The EViews-10 output is the following:

Dependent Variable: D(GDP)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Sample: 1995Q2 2017Q4
Included observations: 91
Convergence achieved after 12 iterations
Coefficient covariance computed using outer product of gradients

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>930.8045</td>
<td>364.6589</td>
<td>2.552535</td>
<td>0.0124</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.215228</td>
<td>0.091947</td>
<td>2.340796</td>
<td>0.0215</td>
</tr>
<tr>
<td>SIGMASQ</td>
<td>4314879.</td>
<td>375234.8</td>
<td>11.49914</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared    0.047224  Mean dependent var  927.4022
Adjusted R-squared 0.025570  S.D. dependent var  2139.875
S.E. of regression 2112.339  Akaike info criterion 18.18191
Sum squared resid  3.93E+08  Schwarz criterion  18.26469
Log likelihood  -824.2770  Hannan-Quinn criter.  18.21531
The AR coefficient is significantly different from zero, but $R^2$ is very small and the correlogram of $d(GDP)$ significantly differ from the estimated correlogram through model figure A1.

Figure A1: Actual and theoretical correlogram for $d(GDP)$

We explain this by saying that $d(GDP)$ is a variable with independent and identically distributed (i.i.d.) values. Indeed, the BDS test applied on $d(GDP)$ series does not reject the i.i.d. hypothesis.

The BDS test is presented in the following table ($\varepsilon = 1.5\sigma$, bootstrap repetitions = 10000, maximum correlation dimension = 4):

**Table A.4. BDS Test for $d(GDP)$**

| Sample: 1995Q1 2017Q4 | Included observations: 92 |
Furthermore, the Variance Ratio Test on d(GDP) does not reject the null hypothesis that d(GDP) is a martingale.

**Table A.5. Variance Ratio Test on d(GDP)**

Null Hypothesis: DGDP is a martingale

Sample: 1995Q1 2017Q4

Included observations: 90 (after adjustments)

Heteroskedasticity robust standard error estimates

User-specified lags: 2 4 8 16

<table>
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Consistent with both tests (BDS and VRT), we do not reject the hypothesis that d(GDP) has not an ARMA structure.