Migrant Labor, Unemployment and Optimal Growth

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Abstract

One of the arguments against migrant labor is that it has negative ef- fect on the employment of domestic labor. The question is now if the immigration has also negative effects on the other variables of the econ- omy. To examine these effects we develop an optimal growth model with migration and unemployment and then we analyze these effects, restricting our analysis to the steady state. We introduce a simplify- ing hypothesis concerning the skill level of human capital. We assume that the average skill level of domestic employed labor differs from the average skill level of migrant labor, but the two kinds of labor grow at the same constant rate. We prove that the immigration process could have both positive and negative effects on consumption, human capital and physical capital, depending on the skill level of the migrant labor. The numerical simulations confirm our theoretical results.

Keywords: immigration; optimal growth; unemployment rate.

JEL Classifications: F2, J6, O4.

Introduction

The phenomenon of migration is present in several developing countries and the existing literature has offered insightful results on the effects of migrant labor. There are many important contributions in this fi Among them we mention the papers of Hazari and Sgro, (2003), Angrist and Kugler (2003), Borjas (2003), Moy and Yip (2006), Dustmann et al. (2008), Fan and Stark (2008), Palivos (2009) and Ottaviano and Peri (2012).

Palivos studied fi the case where there are two types of domestic labor, skilled and unskilled, and second, he introduces a minimum wage, which leads to job competition between domestic unskilled workers and immigrants and, consequently, to unemployment in the domestic labor. The model developed by Palivos is a simply one and it should be viewed as a fi attempt to show that the existence of unemployment can have a significant impact on agents welfare. The key element in its analysis is that he introduces a minimum wage, which applies only to unskilled workers and is assumed to be binding. One of the questionable consequences of the model introduced by Palivos is the fact that an increase in the immigration ratio will leave the capital stock unchanged.

Dustmann, Glitz and Frattini present a stylized model of the labor mar- ket impact of immigration and discuss the mechanisms through which an economy can adjust to immigration. Finally they explain the problems of empirically estimating how immigration affects labor market outcomes of the resident population and review some strategies to address these.

Fan and Stark developed a model of rural-to-urban migration with an emphasis on the role of human capital in both economic activities. Their model assumes that the urban sector produces manufactured goods using labor and physical capital as factors of production, and that the rural sector produces agricultural goods using labor and land as factors of production. This kind of model has been widely used as a basic analytical framework for studying rural-to-urban migration in developing countries and as a platform for policy formation.

In their paper, Angrist and Kugler estimate the effect of migration on native employment in Western Europe countries. Their estimates show that an increase in the foreign share of 10% reduces native employment rates by 0, 2 to 0, 7 of a percentage point. Such an effect may be explained by the fact that there has been little aggregate employment creation in most of Western Europe countries in the last two decades, while immigrant employment has grown considerably.

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In our paper we chose to use the case of the centralized problem, which means a social planner solution via a model of Lucas type. There are many reasons to explain this approach, but at least two reasons have to be mentioned here.

The fi reason is given by the minimum wage. This one is a parameter that characterizes a limited number of economies, and the migration process is a universal process.

The second reason comes from the migration phenomena. As it is well known in many countries the number of migrants is limited by legal regulations. Consequently, the migrant ratio could be considered as a control variable in such a model.

Our approach introduces a simplifying hypothesis concerning the skill level of human capital. We assume that the average skill level of domestic employed labor differs from the average skill level of migrant labor, but the two kinds of labor grow at the same constant rate. This hypothesis can be considered realistic because on the labor market, both domestic and migrant labor follows an educational process to improve their skills. Of course, the alternative hypothesis of different growth rates could be considered, but this one can create some difficulties in the computation procedure. Our fi hypothesis refers to the existence of a constant permanent unemployment rate that is not affected by the migration process.

The rest of the paper is structured as follows. The fi section is this introduction. In the second section we present a model with migrant la- bor and unemployment and in the third and fourth sections we analyze the equilibrium properties in the long run. In the last section, we present some numerical simulations and fi some conclusions.

A growth model with unemployment and migrant labor.

The economic system produces a single commodity Y = F (K, L_{DE}, L_M), under a Cobb-Douglas technology with constant return to scale in physical capital K, domestic employed labor L_{DE} and migrant labor L_M . Under this hypothesis, the production function assumes imperfect substitution between migrant and domestic employed labor since they are considered as separate factors of production.

$$F(K, L_{DE}, L_M) = A_1 K^{\beta} L_{DE}^{\propto} L_M^{\gamma}, \propto + \beta + \gamma = 1$$

Domestic labor is divided into two distinct parts: potentially employed and permanent unemployed $N_D=N_{DPE}+N_{DPU}$. Domestic labor potentially employed consists of two types of labor, skilled and unskilled and write $N_{DPE}=N_{DE}+N_{DUS}$. Migrant labor is a perfect substitute for domes- tic unskilled labor, but not for domestic skilled labor, that is $N_M = N_{DUS}$. Therefore, domestic unemployed labor consists in fact of two distinct types of labor: permanent unemployed and perfect substituted by migrant labor, $N_{DU}=N_{DPU}+N_{DUS}$. Therefore, the effective of total labor employed by the economic $N > NDE$ is given by

$$N = N_{(t)} = N_{DE}(t) + N_M(t)$$

Now we introduce the following notations:

a. Let $u = u(t)$ be the unemployment ratio of domestic labor

$$u(t) = \frac{N_{DU}(t)}{N_D(t)}$$

b. Let $e = e(t)$ be the employment ratio of domestic labor

$$e(t) = \frac{N_{DE}(t)}{N_D(t)} \Longrightarrow e(t) = 1 - u(t)$$

c. Let $u_P = u_P(t)$ be the permanent unemployment ratio of domestic labor

$$\mathrm{u_P}(t) = \frac{N_{DPU}(t)}{N_D(t)}$$

d. Let $u_M = u_M(t)$ be the unemployment ratio generated by migrant labor

$$u_M(t) = \frac{N_M(t)}{N_D(t)}, \Rightarrow u(t) = u_P(t) + u_M(t)$$

e. Let $\omega = \omega(t)$ be the migrant ratio

$$\omega(t) = \frac{N_M(t)}{N(t)}$$

f. Let n be the constant growth rate of $N = N(t)$, that is $n = \frac{N(t)}{N(t)}$

Now we can write:

$$N = N_{DE} + N_M = (1-u)N_D + \omega N \Rightarrow N_D = \frac{1-\omega}{1-u} N \; and \; N_{DE} = (1-\omega)N$$

$$N_D = N_{DE} + N_{DPU} + N_M = (1-\omega)N + u_P N_D + \omega N$$

and finally we get

$$u_P = \frac{u-\omega}{1-\omega} \Leftrightarrow u = u_P + (1-u_P)\omega$$

A similar computation procedure yields

$$u_M = \frac{\omega(_{1-u})}{1-\omega} \Rightarrow u_M + u_P = u$$

We denote by $h_{DE} = h_{DE}(t)$ the average skill level of domestic employed labor, by $h_M = h_M(t)$ the average skill level of migrant labor and by $h = h(t)$ the average skill level of total labor. Hence we can write:

$$L_{DE}(t) = N_{DE}(t)h_{DE}(t), L_M(t) = N_M(t)h_M(t)$$

and without loss of generality we assume the ratio h_M / h_{DE} is constant. This is equivalent to say that:

$$h_M(t) = \mu_M h(t) \; and \; h_{DE}(t) = \mu_D h(t)$$

where μ_M and μ_D are positive constants and thus the production function can be written

$$F = AK^{\beta}N^{1-\beta}h^{1-\beta}\omega^{\gamma}(1-\omega)^{\alpha}, \qquad A = A_1\mu_D^{\alpha}\mu_M^{\gamma}$$

Now we introduce the three main hypotheses of our paper.

1. The first one claims that the ratio of migrant labor is a controlled variable.

2. The second one assumes that the dynamics of the average skill level of total employed labor is described by the following differential equation

$$h = (\delta + \pi\omega)h, \qquad \delta > 0, \qquad \pi \in \mathcal{R}$$

According to this equation, if there are no migrant labor, then $h(t)$ grows at a rate δ. If $\pi > 0$ then the migrant labor has a positive effect on the the growth rate of human capital and if $\pi < 0$ then the migrant labor has a negative effect on the the growth rate of human capital.

3. The third one assumes that the permanent unemployment ratio u_P is constant.

Of course, these parameters could be estimated via econometric methods. In order to simplify the computation procedure we consider all variables as per capita quantities and hence the production function becomes:

$$f = f(k, h, \omega) = Ak^{\beta}h^{1-\beta}\omega^{\gamma}(1-\omega)^{\alpha}$$

Remark 1 *If there are no migrant labor, that is to say $\omega(t) = 0$ for all $t \geq 0$, then we have:* $\gamma = 0, N_M(t) = 0, N(t) = N_D(t), \mu_D = 1,$

$$\lim_{\omega\to 0,\gamma\to 0} \omega^{\gamma} = 1 \; and \; f = f(k,h) = Ak^{\beta}h^{1-\beta}$$

Of course, the two state variables and the two control variables as well as the variable u, are all functions of times, but when no confusions are possible, we simply write *k, h, c, ω* and *u*. Concluding, our model is characterized by the well-known optimization problem.

Definition 1 The set of paths {k, h, c, ω} = {k(t), h(t), c(t), ω(t)} is called an optimal solution if it solves the following optimization problem:

$$V_0 = \max_{c,\omega} \int_0^{\infty} \frac{c^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \tag{1}$$

subject to

$$\begin{cases} k = Ak^{\beta} h^{1-\beta} \omega^{\gamma} (1-\omega)^{\alpha} - c - nk, \\ h = (\delta + \pi\omega)h, \\ k_0 = k(0) > 0, k_0 = h(0) > 0 \end{cases} \tag{2}$$

and

$$u = u_P + \left(1 - u_p\right)\omega, \tag{3}$$

where k_0 and h_0 are given, k is the physical capital, h is the human capital, c is the consumption, β is the elasticity of output with respect to physical capital, α is the elasticity of output with respect to domestic human capital, γ is the elasticity of output with respect to migrant human capital, ρ is a positive discount factor, A > 0, δ > 0, $\pi \in$ IR, and θ^{-1} represents the constant elasticity of intertemporal substitution.

The system (2) gives the resources constraints and initial values for the state variables k and h. To solve the problem (1) subject to (2), we defi the Hamiltonian function (note that unemployment equation doesn't enter):

$$H = \frac{c^{1-\theta} - 1}{1-\theta} + \left[Ak^{\beta} h^{1-\beta} \omega^{\gamma} (1-\omega)^{\alpha} - c - nk\right]\lambda_1 + (\delta + \pi\omega)h\lambda_2$$

The boundary conditions include initial values for human and physical capital and the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t} \lambda_1(t)k(t) = 0 \ \ and \ \ \lim_{t\to\infty} e^{-\rho t} \lambda_2(t)h(t) = 0$$

In this model, there are two control variables, c and ω, and two state vari- ables, k and h, and the optimal trajectory of variable u will be determined as a function of the optimal trajectories of the other variables. In an optimal program the control variables are chosen so as to maximize H. We note that along the optimal path, $\lambda 1$ and $\lambda 2$ are functions of t only. The necessary fi order conditions for the pair (c, ω) to be an optimal control are:

$$\frac{\partial H}{\partial c} = 0 \ \Rightarrow \lambda_1 = c^{-\theta},$$

$$\frac{\partial H}{\partial \omega} = 0 \ \Rightarrow \ \frac{\lambda_1}{\lambda_2} = \frac{\pi\omega(1-\omega)h}{[(1-\beta)\omega - \gamma]f},$$

$$\frac{\lambda_1}{\lambda_1} = \rho + n + \beta\frac{f}{k},$$

$$\frac{\lambda_2}{\lambda_2} = \rho - \delta - \frac{\alpha\pi\omega}{(1-\beta)\omega - \gamma},$$

$$\frac{\omega}{\omega(1-\omega)} = \frac{q(\omega)}{g(\omega)} + \beta\frac{(1-\beta)\omega - \gamma}{g(\omega)}\,\frac{c}{k},$$

where

$$g(\omega) = \omega\beta(1-\beta)\omega^2 - 2\gamma\beta\omega + \gamma(1-\gamma)$$

and

$$g(\omega) = (1-\beta)\{\pi\beta\omega^2 - [\pi(1-\gamma) + (1-\beta)(\delta + n)]\omega + \gamma(n + \delta)\}$$

First observe that $g(0) = \gamma(1 - \gamma) > 0$, $g(1) = \alpha(1 - \alpha) > 0$ and since the discriminant $\Delta_g = -\alpha\beta\gamma < 0$ we conclude that $g(\omega) > 0$ for all $\omega \in [0, 1]$. For the function q we have. $q(0) = \gamma(1-\beta)(n+\delta) > 0$, $q(1) = -\alpha(1-\beta)(\pi+n+\delta)$ and the discriminant Δ_q is given by

$$\Delta_q = (\alpha+\gamma\beta)^2\pi^2 + 2(\delta+n)(\alpha-\gamma\beta)(1-\beta)^2\pi + (1-\beta)^4(\delta+n)^2$$

Δ_q as a function of π is always positive because $(\alpha + \gamma\beta)^2 > 0$ and its discriminant $\Delta = -4\alpha\gamma(1 - \beta)^4(\delta + n)^2$ is always negative. Consequently, if $\pi > -n - \delta$ then there exists a unique $\omega_0 \in [0, 1]$ such that $q(\omega_0) = 0$ and, for all $\omega \in [0, \omega_0]$ the function $q(\omega) > 0$ and for all $\omega \in [\omega_0, 1]$ the function $q(\omega) < 0$. Of course, if $\pi < -n - \delta$ then there exist two real roots outside of the interval (0, 1) and the function $q(\omega) > 0$ for all $\omega \in [0, 1]$. After some algebraic manipulations, we can close the system and write down the final form

$$\begin{cases} \dfrac{\dot{k}}{k} = A\left(\dfrac{h}{k}\right)^{1-\beta}\omega^\gamma(1-\omega)x^\alpha - n - \dfrac{c}{k}, \\ \dfrac{\dot{h}}{h} = \delta + \pi\omega, \\ \dfrac{\dot{c}}{c} = \dfrac{\beta A}{\theta}\left(\dfrac{h}{k}\right)^{1-\beta}\omega^\gamma(1-\omega)^\alpha - \dfrac{\rho+n}{\theta}, \\ \dot{\omega} = \dfrac{\omega(1-\omega)}{g(\omega)}\left\{q(\omega) - \beta[\gamma-(1-\beta)\omega]\dfrac{c}{k}\right\}, \\ \dfrac{\dot{\lambda}_1}{\lambda_1} = \rho + n - \beta A\left(\dfrac{h}{k}\right)^{1-\beta}\omega^\gamma(1-\omega)^\alpha, \\ \dfrac{\dot{\lambda}_2}{\lambda_2} = \rho - \delta - \dfrac{\alpha\pi\omega}{(1-\beta)\omega-\gamma}, \end{cases}$$

and

$$u = u_P + (1-u_p)\omega$$

The balanced growth path

In this section we examine the properties of the balanced growth path (BGP). The system described above reaches the balanced growth path if there exists a finite $t_* > 0$, such that for all $t \geq t_*$, $r_\omega = 0$ and $r_k = r_c = r_h$, where r_x denotes the growth rate of variable x, x_* is its value at $t = t_*$ and x^* is its value for $t > t_*$. The following proposition gives our fi result that characterize the balanced growth path.

Proposition 1 Let $\pi \in \mathbb{R}$ and $\theta > 1$. If for all $t \geq t_*$, $r\omega = 0$, then the above system reaches the BGP and the following statements are valid

i. $r_{f*} = r_{k*} = r_{c*} = r_{h*} = r_* = \delta + \pi\omega_*$

ii. There exists at least one $\omega_* \in [0, 1]$, solution of the equation $A_0\omega^2 + A_1\omega + A_2 = 0$, given by

$$\omega_* = \frac{-A_1 \pm \sqrt{A_1^2 - 4A_0A_2}}{2A_0}$$

where

$$A_0 = \omega\theta(1-\beta), \qquad A_2 = -\gamma[\delta(\theta-1)+\rho]$$
$$A_1 = (1-\beta)[\delta(\theta-1)+\rho] - \pi(\alpha+\gamma\theta)$$

iii.

$$u_* = u_P + (1 - u_p)\omega_*$$

iv.

$$\frac{c_*}{k_*} = \frac{(\theta - \beta)r_* + \rho + n(1 - \beta)}{\beta}$$

v.

$$\frac{f_*}{k_*} = \frac{\theta r_* + \rho + n}{\beta}$$

vi.

$$\frac{h_*}{k_*} = \left\{\frac{\theta r_* + \rho + n}{\beta A} \frac{1}{(1 - \omega_*)^{\alpha} \omega_*^{\gamma}}\right\}^{\frac{1}{1-\beta}}$$

Proof of Proposition 1. At (*BGP*) ω is constant and therefore $\omega = 0$˙ From the fourth equation of the system (4) we obtain

$$\frac{c_*}{k_*} = \frac{q(\omega_*)}{\beta[\gamma - (1 - \beta)\omega_*]}$$

This is equivalent to say that there exists a common constant growth rate for both variables *c* and *k*, for any $t > t_*$. Let us denote by r_* this common and constant growth rate. The fi equation of the system (4) can then be written

$$r + n + \frac{c}{k} = A\left(\frac{c}{k}\right)^{1-\beta} \omega^{\gamma}(1 - \omega)^{\alpha}$$

At *BGP* , the left side of the above relation and ω are both positive real constants. Taking the logarithm and then diff tiating with respect to time we obtain

$$(1 - \beta)(r_h - r_k) = 0 \implies r_h = r_k = r_*$$

Now, the common growth rate can be determined from the second equation of the system (4) and is given by $r_* = \delta + \pi\omega_*$. Combining the fi and the third equation of the system (4) we get

$$r_* + \frac{\rho + n(1 - \beta)}{\theta - \beta} = \frac{\beta}{\theta - \beta} \frac{c_*}{k_*}$$

from where it follows

$$\frac{c_*}{k_*} = \frac{\theta - \beta}{\beta}(\delta + \pi\omega_*) + \frac{\rho + n(1 - \beta)}{\beta}$$

Equalizing the two results from the equations (10) and (11) we obtain

$$G(\omega_*) = A_0\omega_*^2 + A_1\omega_* + A_2 = 0$$

The discriminant Δ_G of the function $G(\omega_*)$ is given by

$$\Delta_G = (\alpha + \gamma\theta)^2\pi^2 + 2(1 - \beta)(\gamma\theta - \alpha)[\delta(\theta - 1) + \rho]\pi + (1 - \beta)^2[\delta(\theta - 1) + \rho]^2$$

Δ_G as a function of π is always positive because $(\alpha+\gamma\theta)^2 > 0$ and its discrim- inant $\Delta = -4\alpha\gamma\theta(1 - \beta)^2 [\delta(\theta - 1) + \rho]^2$ is always negative. Consequently, because $G(0) = -\gamma [\delta(\theta - 1) + \rho] < 0$ and $G(1) = \alpha [(\pi + \delta)(\theta - 1) + \rho]$, we can distinguish here two possibilities:

1. If $\pi > -\delta - \frac{\rho}{\theta - 1}$ then $(\pi + \delta)(\theta - 1) + \rho >$, and we deduce that there exist two real solutions, one solution $\omega_1 \in [0, 1]$ and one solution $\omega_*^2 \notin [0, 1]$, such that $G(\omega_*^1) = G(\omega_*^2) = 0$given by (5).

2. If $\pi < -\delta - \frac{\rho}{\theta-1}$, then then $(\pi + \delta)(\theta - 1) + \rho < 0$ and we deduce that there exist two solutions $\omega_* \in [0,1]$ such that $G(\omega_*) = 0$ given by (5).

The last two results follow immediately by direct computation and thus the proof is completed.

Short analysis of the BGP

At this stage, a short analysis is absolutely necessary. As we can observe from the above relations, the values at steady state depend only on the parameters of our economy. This claim is obviously true since the optimal level of the migration rate - given by the relation (5), depends only on these parameters and all other optimal values of the economy are affected by the optimal level of the migration rate. As we pointed out in the introduction section, one of the arguments against immigration is that it increases the unemployment of the domestic labor. Unfortunately it is too diffi to analyze this effect for the transitional dynamics and therefore we restrict our analysis only to the steady state. The question is now which are the effects of immigration on the other variables. The following proposition tries to give a coherent answer to this question.

Proposition 2 The balanced growth path determined in the previous section has the following properties.

1. For any real value of the efficiency parameter of migrant labor π, the immigration has negative effects on the unemployment rate.
2. If the efficiency parameter of migrant labor $\pi > 0$, then the immigration has positive effects on all the other variables of the economy.
3. If the efficiency parameter of migrant labor $\pi < 0$, then the immigration has negative effects on all variables of the economy.

Proof of Proposition 2. First we introduce the following functions.

$$P_1(\omega_*) = u_*, \qquad P_2(\omega_*) = ur_*, \qquad P_3(\omega_*) = \frac{c_*}{k_*}, \qquad P_4(\omega_*) = \frac{f_*}{k_*}, \qquad P_5(\omega_*) = \frac{h_*}{k_*}$$

Taking now the derivative of each function with respect to ω_*, denoted by $P_k^{`}, k = 1 \div 5$, we obtain

$$P_1^{`}(\omega_*) = 1 - u_P > 0, \qquad P_2^{`}(\omega_*) = \pi, \qquad P_3^{`}(\omega_*) = \frac{(\theta - \beta)\pi}{\beta}, \qquad P_4^{`}(\omega_*) = \frac{\theta\pi}{\beta}$$

$$P_5^{`}(\omega_*) = \frac{P_5(\omega_*)}{(1-\beta)\omega_*(1-\omega_*)} \frac{C_0\omega_*^2 + C_1\omega_* + C_2}{\theta\pi\omega_* + \delta\theta + \rho + n}$$

Where

$$C_0 = -\omega\theta\beta, \qquad C_2 = -\gamma(\theta\delta + \rho + n)$$
$$C_1 = (1-\beta)(\theta\delta + \rho + n) + \pi\theta(1-\gamma)$$

Let us we denote by $P(\omega) = C_0\omega^2 + C_1\omega + C_2$. The discriminant Δ_P of the function P (ω) is given by

$$\Delta_P = \theta^2(1-\gamma)^2 - 2\theta(\alpha - \gamma\beta)(\delta\theta + n + \rho)\pi + (1-\beta)^2(\delta\theta + n + \rho)^2$$

Δ_P as a function of π is always positive because $\theta^2(1-\gamma)^2 > 0$ and its discriminant $\Delta = -4\alpha\gamma\beta\theta^2(\delta\theta + \rho + n)^2$ is always negative. Consequently, because

$$P(0) = -\gamma(\theta\delta + \rho + n) < 0 \; and \; P(1) = \alpha[\rho + n + \theta(\delta + \pi)]$$

we can distinguish here two possibilities:

1. If $\pi \in (-\delta - \frac{\rho+n}{\theta}, 0)$ then $\rho + n + \theta(\delta + \pi) > 0$, and we deduce that there exist two real solutions, one solution $\omega_P \in [0, 1]$ and one solution $\omega^2 \notin [0,1]$. Therefore, for all $\omega \in (\omega_P,1)$ the function P (ω) < 0, the function $\theta\pi\omega_* + \delta\theta + \rho + n > 0$ and consequently the function$P_5^{`}(\omega_*) < 0$.

2. If $\pi > 0$ then $\rho + n + \theta(\delta + \pi) > 0$, and we deduce that there exist two real solutions, one solution $\omega_P \in [0, 1]$ and one solution $\omega^2 \notin [0,1]$. Therefore, for all $\omega \in (\omega_P,1)$ the function P (ω) > 0, the function $\theta\pi\omega_* + \delta\theta + \rho + n > 0$ and consequently the function $P_4^{`}(\omega_*) > 0$.
3. If $\pi < (-\delta - \frac{\rho+n}{\theta}, 0)$, then then $\rho + n + \theta(\delta + \pi) < 0$ and we deduce that there exist two solutions $\omega \notin [0,1]$. Therefore, for all $\omega \in [0,1]$ the function P (ω) < 0, the function $\theta\pi\omega_* + \delta\theta + \rho + n > 0$ and consequently the function $P_5^{`}(\omega_*) < 0$

and thus the proof is completed.

Concluding, we may claim that the immigration process could have both positive and negative effects on consumption, human capital and physical capital, depending on the skill level of the migrant labor. These results con- tradict those obtained by Palivos, where it was shown that, if expanded, in a rather simple way, to allow for unemployment in the labor force, consumption and welfare decrease.

Conclusions and numerical simulations

As we pointed out above, one of the arguments against immigration is that it increases the unemployment of the domestic labor and has negative effects on all the other variables of the economy. The main aim of this fi section is to confirm by numerical simulations the theoretical aspects presented in this paper. In order to do this we close this section presenting the results of a numerical simulation procedure. The benchmark values for economy we consider are the following:

a. $\beta = 0.25$, $n = 0.01$, $\delta = 0.15$, $\pi = 0.05$, $\rho = 0.04$, $\alpha = 0.60$, $\gamma = 0.15$, $\theta = 1.2$, $A = 1.05$ and the corresponding steady state equilibrium is given by:

$\omega_* = 0.0728$, $u_* = 0.0821$, $r_* = 0.1536$,

$$\frac{h_*}{k_*} = 1.5424, \qquad \frac{c_*}{k_*} = 0.7738, \qquad \frac{f_*}{k_*} = 0.9375$$

b. $\beta = 0.25$, $n = 0.01$, $\delta = 0.15$, $\pi = -0.05$, $\rho = 0.04$, $\alpha = 0.60$, $\gamma = 0.15$, $\theta = 1.2$, $A = 1.05$ and the corresponding steady state equilibrium is given by:

$\omega_* = 0.0690$, $u_* = 0.0783$, $r_* = 0.1465$,

$$\frac{h_*}{k_*} = 1.4791, \qquad \frac{c_*}{k_*} = 0.7469, \qquad \frac{f_*}{k_*} = 0.9034$$

Under the above baseline of the parameters, the model conform roughly to standard empirical evidence and to other results obtained by the above cited authors. The numerical simulation confirm our theoretical results. As we can observe, the two coefficients δ and π play a crucial role in the effects of the migration process. As we claimed above, the three coefficients δ, π and uP could be estimated only via econometric methods.

In this paper we have examined the existence and some properties of the

balanced growth path of a model with migrant labor and unemployment under a Cobb-Douglas production technology. We have also proved that the immigration process could have both positive and negative effects on consumption, human capital and physical capital.

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