Nonlinear Models for Economic Forecasting Applications: An Evolutionary Discussion

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Abstract

This article follows the main contributions brought to the nonlinear modeling literature. We investigate and review a series of parametric initiatives, focusing on the evolution of TAR and ARCH – GARCH model families in econometric and forecasting applications.

Keywords: nonlinear parametric models, threshold models, ARCH - GARCH models

1. Introduction

Around the 1980's, the modeling of the nonlinear dynamics becomes one of the most popular methodologies in the study of financial markets, macroeconomics, regional studies and environmental issues. The growth of this approach was based on various motivations that circled around technical aspects such as the limitations of the neoclassic models that seemed incapable of capturing certain features of the economic reality and the weak results offered by the traditional linear stochastic models.

In spite of this, the first attempts towards nonlinear modeling date back to the end of the Great Depression. The same moment saw the expansion of another field, macroeconomics, which was focusing on business cycles and their fluctuations. The dynamic analysis came to be deeply rooted into the expansion of macroeconomics to such an extent that [1] argued about the appearance of a new research field: macrodynamics.

The origins of econometric nonlinear modeling can be traced back to the pioneering work of [2]. A few years earlier, [3] had launched a business cycle model that incorporated a time lag between a decision on an investment and its subsequent effect on capital stock. [2] builds on these results and succeeds in explaining fluctuations by the use of nonlinearity. Since these early attempts, a strong and very active literature emerged. Given the dimension of the topic, the purpose of this paper is to concentrate only on the parametric nonlinear models.

2. The Threshold Model Class

One very extensively used class of nonlinear time series models is the TAR (threshold autoregressive) model class brought forward by [4] and later refined in [5]. The basic idea behind these models consists in linear approximations of subspaces of the initial space. This segmentation is done by the use of a threshold variable.

The most important features of the TAR models are their simplicity and versatility, but in spite of this schematic nature, they are able to generate a complex image of the nonlinear dynamics. The main limitation of the TAR models derives from the large number of parameters that need to be estimated in their construction.

By definition, a TAR model with *n* regimes has the following form:

$$X_t = \sum_{i=1}^{n} \{b_{i0} + b_{i1}X_{t-1} + \cdots + b_{i,pi}X_{t-pi} + \sigma_i \varepsilon_t\} I(X_{t-d} \in R_i) \tag{1}$$

Where:

$\{\varepsilon_t\} \sim IID(0,1)$,

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$d\ and\ p_1, \cdots, p_n$ represent a series of unknown positive, variables

$\sigma_i > 0\ and\ b_{i,j}$ represent unknown parameters

And $\{R_i\}$ is a partition of the $(-\infty, \infty)$ interval so that:

$U_{i=1}^{n} R_i = (-\infty, \infty)$ and $R_i \cap R_j = \emptyset$, for any given $i \neq j$.

Due to their general tractability, TAR models became popular and saw a wide use in many fields of modern economics, ranging from macroeconomics to finance.

An example of the use of TAR models in macroeconomics is [6]. In their study on recessions and output they offer a modeling scheme that considers GNP rates of growth as a function of the deviation of the current GNP from the historical maximum values of this parameter.

[7] argue that the above model is actually a particular form of TAR model and extend the logic to incorporate floor and ceiling effects.

While studying the US unemployment rate, [8] test the performance of several time-series models and conclude that the TAR models outperform linear models during contraction intervals.

TAR models are also extensively used in the study of financial markets. Addressing the problem of modeling the difference in princes of equivalent assets, [9] use TAR models and discuss the statistical estimation and testing procedures. Using the example of an index futures contract and the equivalent cash index, the authors clearly reject the linearity hypothesis and observe the threshold nonlinearity.

By combining TAR modeling with an error-correction component, [10] show the impact in mispricing for intraday futures and index returns. This approach allows the authors to model the behavior of arbitragers.

In the modeling context of equation (1), each R_i can be set to a linear form. The segmentation is given by the threshold variable X_{t-d}, where *d* stands for a delay parameter. Thus, $R_i = (r_{i1}, r_i]$, where where $-\infty = r_0, < r_1 < \cdots < r_n = \infty$, and the r_i variables represent the thresholds. As observed by [11], in this case the original TAR model turns into self-exciting threshold model (SETAR).

Generating from the work of Tong from 1977, SETAR became quickly popular in a wide range of econometric applications.

Assuming the $R_1, \cdots, R_l$ intervals as to allow $R_1 \cup \cdots \cup R_l = \mathbb{R}$ and $R_i \cap R_j = \emptyset\ \forall i, j$, each interval R_i is expressed as $R_i =]r_{i-1}; r_i]$, where $r_0 = -\infty, r_1, \cdots, r_{l-1} \in R$, and $r_l = \infty$. Under these assumptions, [12] draw the standard form for the SETAR (l; d;$k_1, k_2, \cdots, k_l$) model as:

$$X_t = a_0^{(J_t)} + \sum_{i=1}^{k_{J_t}} a_i^{(J_t)} X_{t-i} + \epsilon_t^{(J_t)} \tag{2}$$

Where:

$$J_t = \begin{cases} 1 & X_{t-d} \in R_1 \\ 2 & X_{t-d} \in R_2 \\ & \cdots \\ l & X_{t-d} \in R_l \end{cases} \tag{3}$$

[13] investigate the nonlinear dynamics of short-term interest rate in the US using a SETAR model and evaluate the performance of the model through the point of view of the term structure.

[14] study SETAR models on macroeconomic time series and conclude that these are more eficient in forecasting in comparison to AR models. On the contrary, [7] report that the AR models are more efficient in predicting the conditional mean but not also in terms of variance.

[15] report that SETAR models are more efficient than AR models in certain regimes, though this characteristic isn't relevant throughout the entire data set.

[16] also conduct a performance test involving SETAR, AR and GARCH models, on data sets representing the exchange course of the EURO. The authors conclude that at an aggregate level, the GARCH model is the most performant in capturing the properties of the time series.

If the SETAR models involve a finite number of regimes, [17] proposes a model for perpetual regimes, called the Smooth Transition Autoregressive (STAR) model. The general form of STAR models according to [18] is:

$$y_t = \phi_0 + \sum_{i=}^{p} \phi_i y_{t-i} + \left(\theta_0 + \sum_{i=1}^{p} \theta_i y_{t-i} \right) F(y_{t-d}) + \varepsilon_t \tag{4}$$

where $F(y_{t-d})$ represents a continuous transition function that can be either logistic or exponential.

$$F(y_{t-d}) = \left[1 + exp\left(-\gamma(y_{t-d} - r)\right)\right]^{-1} \tag{5}$$

$$F(y_{t-d}) = 1 - \left[exp\left(-\gamma(y_{t-d} - r)\right)^2\right] \tag{6}$$

These transition functions determine the type of STAR model used: LSTAR (logistic STAR) or ESTAR (exponential STAR).

One example of the use of LSTAR models is [19]. The authors investigate various time series that characterize the business cycle and observe the nonlinear characteristics of these data sets. ESTAR models have been applied in finance in the study of exchange rates [20] or [21].

One of the most used applications of nonlinear TAR models is the vector error-correction model (VECM). This is actually a mix between the classical TAR background and the model of cointegration of Engle and Granger [22] and was put forward by [23].

Another interesting solution was introduced by [24] and represents the vector TAR (VTAR) model.

The theoretical literature that builds on these modeling initiatives is extensive, key studies having been brought forward by [25], [26] or [27].

Vector error-correction models have been extensively used in the study of the dynamics of financial markets. [28] use a VECM to obtain monthly stock market levels and monthly stock returns for the case of the Singapore market.

In an investigation on the existence of long-run equilibrium between stock prices and industrial production, real exchange rate, interest rate, and inflation for the United States, [29] uses a VECM model similar to [28] and reports that the S&P 500 price is positively related to the industrial production but negatively to the rest of the variables.

[30] build on the VECM model, and use their methodology on a set of U.S. disaggregated CPI data. They find evidence of threshold cointegration especially for tradable goods.

There is an important number of generalizations or special cases of TAR models that circulated in the literature. One of these models is the Open Loop Threshold AR (TARSO), launched by [31] which uses an exogenous input time series. The general form of the model is the following:

$$X_t = a_0^{(J_t)} + \sum_{i=1}^{k_{J_t}} a_i^{(J_t)} X_{t-i} + \sum_{i=1}^{k'_{J_t}} b_i^{(J_t)} U_{t-i} + \epsilon_t \tag{7}$$

As observed by [12], in this case the regimes shifts are determined by:

$$J_t = \begin{cases} 1 & U_{t-d} \in R_1 \\ 2 & U_{t-d} \in R_2 \\ & \dots \\ l & U_{t-d} \in R_l \end{cases} \tag{8}$$

TARSO models have been used in estimations on stock returns and real economic activity ([32], [33]) and in forecasting spot prices [34]

Another generalization of the SETAR model is the self-exciting Threshold ARMA (SETARMA) which is an obvious extension given by the following equation.

$$\boldsymbol{X_t = a_0^{(J_t)} + \sum_{i=1}^{k_{J_t}} a_i^{(J_t)} X_{t-i} + \sum_{i=1}^{k'_{J_t}} b_i^{(J_t)} \epsilon_{t-i} + \epsilon_t} \qquad (9)$$

3. The ARCH – GARCH model class

ARCH models (AutoRegressive Conditional Heteroscedasticity) had their genesis in the study conducted by [35] which sought to model variances for British inflation rates and in the meantime became a common and apreciated solution for the investigation of volalitilities.

The original ARCH (q) model as proposed by [35] has the following form:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 = \omega + \alpha(L)\varepsilon_t^2 \qquad (10)$$

Where $\omega > 0, \alpha_i \geq 0$ and L represents the lag operator.

This linear setup is very useful in financial aplication as it holds the tendency for volatility clustering, meaning the tendancy that price changes to be followed by other price changes of an unpredictable sign.

A more popular alternative to the above model is the Generalizez ARCH or GARCH(p,q) developed by [36] and [37].

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \qquad (11)$$

The characteristic that made the GARCH (p,q) so popular is the dependence in ε_t^2.. The equaton above can be easily translated as an ARMA model for ε_t^2 with the autoregressive aparameters *α(L)+β(L)* [38].

In the above GARCH (p,q) specification, the variance depends on the size, but not also on the sign of ε_t. This fact is inconsistent with the actual evolution of financial data. This shortcoming was fixed by [39] who treats σ_t^2 as an asymmetric function of the evolution of the values of the past ε_t:

$$log\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i(\phi z_{t-i} + \gamma[|z_{t-i}| - E|z_{t-i}|]) + \sum_{i=1}^{p} \beta_i log\sigma_{t-i}^2 \qquad (12)$$

Unlike its linear precursors the EGARCH model imposes no restrictions on the sign of the conditional variances.

The scientific literature abounds in nonlinear ARCH alternatives such as the models of [40] and [41].

Another interesting innovation was the GJR-GARCH model established by [42]. Its distinctive characteristic was the fact that it captured the asymmetric effects of shocks (both positive and negative), which is a useful aspect in the study of the leverage effect.

The general form of the model can be described as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{m} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{s} \alpha_j a_{t-j}^2 + \sum_{j=1}^{s} \gamma_j I_{t-j} a_{t-j}^2 \qquad (13)$$

[43], and [44] use smooth transitions between regimes in order to obtain a nonlinear version of the GJR-GARCH model. This resulted in the Logistic Smooth Transition GARCH (LSTGARCH(1,1)) model defined as:

$$\sigma_t^2 = w + \left(1 - F(\varepsilon_{t-1})\right)\alpha_1\varepsilon_{t-1}^2 + F(\varepsilon_{t-1})\gamma_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \tag{14}$$

where F represents the transformation function. If F is an exponential function as depincted by [43], the model becomes an Exponential Smooth Transition GARCH (ESTGARCH). [45] also build on this idea incorporating a continous bounded function.

[46] develop a nonlinear threshold model called Double Threshold ARCH (DTARCH). In this case both the autoregressive conditional mean and the conditional variance are built on a threshold patterns. Threshold ARCH models originated from the work of [47] and assume that the conditional standard deviation is a function of the value of the shocks.

Other nonlinear ARCH - GARCH models with interesting properties are: the Asymmetric Power ARCH (A-PARCH - [48]), the Volatility Switching GARCH (VSGARCH [49], the Asymmetric Nonlinear Smooth Transition GARCH (ANST-GARCH - [50] (1999)), the Quadratic GARCH (QGARCH – [51]), or the Markov-Switching GARCH (MSW-GARCH – [52]).

4. Conclusions

Nonlinear parametric models have been very successful in the analysis of a wide range of economic fields, ranging from macroeconomics to financial markets. In this paper we have aimed to review a selection of the existing literature on nonlinear models that are used in econometric and forecasting application. We focused only on the parametric models and especially on the TAR and ARCH – GARCH families of models

The survey was concentrated on the updates brought by each modeling initiative, from an evolutionary perspective.

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