

# The long-term causality. A comparative study for some EU countries.

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## Abstract

*Confronted with the inadequacies of the macro econometric models of Keynesian inspiration, Sims (1980) formulates the famous criticism of Sims and proposes a multivariate modelling, where the only limitations are the choice of the selected variables and the number of integrated delays. An alternative to this formulation is the starting point of this article namely, only statistical data can confirm a theory. As it is well-known, the endogenous growth models usually examine all kinds of dependencies between macroeconomic variables. In this paper, we propose an analytical approach of some of these dependencies via the VAR approach, in order to put in evidence the causal effect and to do a comparative study of three EU countries Germany, France and Romania. The obtained results widely confirm the theoretical hypotheses of the endogenous growth models.*

**Keywords:** *Vector Autoregressive model, economic growth, Causality Test.*

**JEL classification:** *C01, C32, C51.*

## 1. Introduction

The question I will try to give an answer in this paper is: Does investment in education necessarily enhance economic growth? There are compelling reasons that it should, but the empirical evidence does not always support this conclusion, as it follows from the paper of

Benhabib and Spiegel (1994). Other studies especially those realized by Psacharopoulos (1993) and Carnoy (1995), essentially show that there exists a positive relation between an individual's level of education, his or her productivity, and his or her earnings.

The macroeconomic analyses of growth appeared at the end of the 1990s, with the paper of Barro and Salla-i-Martin (1991), within a convergence framework. They were the first to show that, for a given level of wealth, the economic growth rate was positively related to the initial level of human capital of a country, whereas for a given level of human capital, the growth rate was negatively related to the initial level of GDP per capita. Convergence, therefore, appears to be strongly conditioned by the initial level of education. One year later, Mankiw, Romer, and Weil (1992) assume that the level of saving, demographic growth and investment in human capital determine a country's stationary state. They also find that these different stationary states seem to explain the persistence of development disparities. Consequently, these different studies show that the variations of growth rates among countries can be explained partly by the initial level of human capital.

However, can we claim that a higher level of investment in education affects the growth path? That is, in terms of the economic convergence analyzed by Barro and Salla-i-Martin (1991), could investments in education modify the transitional path to equilibrium? It is difficult to formulate an answer to this question using only econometric techniques. As I mentioned above, it is this attempt to estimate the macroeconomic relation between investment in education and output that produces major contradictions. For this reason, my analysis requires two different approaches: the VAR model introduced by Christopher Sims in the early 1980s and the concept of causality.

The concept of causality was initially introduced by Wiener (1956) and later by Granger (1969) and constitutes a basic notion for studying dynamic relationships between time series. This concept is defined in terms of predictability at horizon one of a (vector) variable  $X$  from its own past and the past of another (vector) variable  $Y$ . Granger gave a very simple definition of the causality, which can be easily tested by econometric techniques.

**Definition 1.** We say that the variable  $y_t$  causes the variable  $x_t$ , if the predicted error variance of the variable  $x_t$  obtained using both its past and the past of the variable  $y_t$  is lower than the forecast error variance of variable  $x_t$  obtained by knowing only its past:

$$\sigma_{\epsilon}^2(x_t | x_{t-1}, x_{t-2}, \dots, y_{t-1}, y_{t-2}, \dots) \leq \sigma_{\epsilon}^2(x_t | x_{t-1}, x_{t-2}, \dots).$$

This theory, known our days as the theory of Wiener-Granger causality has generated a considerable literature. We mention here only reference work in the field of Geweke (1982).

For the case of a bivariate VAR model, the analysis of Wiener-Granger distinguishes among three types of causality: two unidirectional causalities (called feedbacks) from X to Y and from Y to X and an instantaneous causality associated with contemporaneous correlations. In practice, it is possible that these three types of causality coexist, hence the importance of finding means to measure their degree and determine the most important ones. Unfortunately, existing causality tests fail to accomplish this task, because they only inform us about the presence or the absence of causality. Geweke extended the causality concept by defining measures of feedback and instantaneous effects, which can be decomposed in time and frequency domains. This measure has been determined for a time horizon equal to unity and can not capture indirect causal effect, i.e. when an auxiliary variable Z does not influence directly the variable X, but indirectly through the variable Y.

Another issue that requires to be studied is the persistence of causality, or simply the manner in which it is transmitted. To understand this one we examine the following example. Let us consider the following stationary bivariate VAR(1) model:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0.50 & 0.70 \\ 0.40 & 0.35 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix} \quad (1)$$

so that  $x_t$  is given by the equation:

$$x_t = 0.50x_{t-1} + 0.70y_{t-1} + u_t. \quad (2)$$

Since the coefficient of  $y_{(t-1)}$  in equation (2) is equal to 0.7, we can conclude that Y causes X in the sense of Granger. However, this does not give any information on causality at horizons larger than one nor on its strength. To study causality at horizon two, consider the system (1) at time  $t + 1$  and obtain:

$$\begin{bmatrix} x_{t+1} \\ y_{t+1} \end{bmatrix} = \begin{bmatrix} 0.530 & 0.595 \\ 0.340 & 0.402 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} 0.50 & 0.70 \\ 0.40 & 0.35 \end{bmatrix} \begin{bmatrix} u_t \\ v_t \end{bmatrix} + \begin{bmatrix} u_{t+1} \\ v_{t+1} \end{bmatrix}. \quad (3)$$

In particular,  $x_{(t+1)}$  is given by

$$x_{t+1} = 0.530x_{t-1} + 0.595y_{t-1} + 0.50u_t + 0.70v_t + u_{t+1}. \quad (4)$$

The coefficient of  $y_{(t-1)}$  in equation (4) is equal to 0.595, so Y causes X at horizon two. But, how can one measure the importance of this long-run causality? Existing measures do not answer this question. Nevertheless, recent approaches have succeeded to clarify this issue, in a very simple manner. I mention in particular here the contributions of Dufour and Renault (1998), Dufour and Taamoutic (2010) who managed to define a causality measure at a time horizon  $h > 0$ , and those of Stern and Enflo (2013) who applied these techniques to study the effect of energy consumption on economic growth.

This paper has four sections. The first one is this introduction. The second section is dedicated to developing a VAR model which aims to highlight causality. Section three studies the causality between GDP per capita and investments allocated to education, and the last section presents some conclusions.

## 2. A measure of causality - a VAR approach

Education plays a crucial role in creating human capital, which contributes to production and economic growth just as physical capital and labor do. One of the arguments in support of the conclusion that investment in education does contribute to growth is that almost all countries with high level of economic growth have labor forces with high level of education. This standard of education was obviously obtained as a result of resources

allocated to education. On the other hand, it can also apparently be said that investment in education were in turn substantially determined by the level of development of each country.

Can we argue all these conclusions resulting from statistical observations by using mathematical models? If the answer is positive, for example, by using various statistical tests, then we can claim that the theoretical aspects correspond widely with the reality, as expected Sims. By doing in this way, we can obviously justify some of the theoretical aspects concerning the development of the endogenous growth models.

The starting point of the analysis I intend to develop in this paper is the model introduced by Dufour and Taamouti. I consider the case of a stationary bivariate VAR(1) model, where the two variables are: GDP per capita and investment in education, denoted here by X and by Y.

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \varphi_{xx} & \varphi_{xy} \\ \varphi_{yx} & \varphi_{yy} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix} \quad (5)$$

where  $u_t$  and  $v_t$  are uncorrelated white noise stochastic processes with zero means and constant variances, sometimes called innovations. The assertion that the model is stationary, is actually equivalent to the claim that the absolute value of the roots of the lag polynomial

$$\det[\Phi(L)] = \det[I_2 - \Phi L]$$

are all superior to one.  $\Phi L$  simply signify the multiplication of matrix  $\Phi$  with variable L. Let denote by  $\Phi^*(L)$  the adjoint matrix of the matrix  $\Phi(L)$ , given by:

$$\Phi^*(L) = \begin{bmatrix} 1 - \varphi_{yy}L & \varphi_{xy}L \\ \varphi_{yx}L & 1 - \varphi_{xx}L \end{bmatrix} \quad (6)$$

then we obviously have

$$\Phi^*(L)\Phi(L) = \det[\Phi(L)]I_2 \quad (7)$$

and finally obtain:

$$\text{Det}[\Phi(L)] = 1 - (\varphi_{xx} + \varphi_{yy})L - (\varphi_{xy}\varphi_{yx} - \varphi_{xx}\varphi_{yy})L^2. \quad (8)$$

In matrix form, the model (5) can also be written:

$$X_t = \Phi X_{t-1} + U_t, \quad \Phi = \begin{bmatrix} \varphi_{xx} & \varphi_{xy} \\ \varphi_{yx} & \varphi_{yy} \end{bmatrix}, \quad X_t = \begin{bmatrix} X_t \\ Y_t \end{bmatrix}, \quad U_t = \begin{bmatrix} u_t \\ v_t \end{bmatrix} \quad (9)$$

Under stationarity,  $X_t$  is characterized by the following autoregressive moving average representation VMA( $\infty$ ):

$$X_t = \sum_{j=0}^{\infty} \psi_j U_{t-j}, \quad \text{where } \psi_j = \Phi^j \text{ si } \psi_0 = \Phi^0 = I_2. \quad (10)$$

The results obtained in this paper, use the lemma 5.1 of the paper of Dufour and Taamouti and for that reason I shall give here a short presentation.

Let  $u_t$  be a bidimensional white noise process with nonsingular variance-covariance matrix  $\Sigma_u$  and let

$$W_t = \mu + \sum_{j=1}^q \psi_j u_{t-j} + u_t \quad (11)$$

be a bidimensional invertible VMA(q) process. Furthermore, let  $F = [1 \ 0]$ . Then the one dimensional process  $V_t = FW_t$ , has an invertible VMA( $\bar{q}$ ) representation:

$$V_t = F\mu + \sum_{j=1}^{\bar{q}} \bar{\theta}_j \varepsilon_{t-j} + \varepsilon_t, \quad \bar{q} < q \quad (12)$$

where  $\varepsilon_t$  is one dimensional white noise with nonsingular variance  $\sigma_\varepsilon^2$ , and  $\theta_j, j = 0, 1, \dots, \bar{q}$  are constant coefficients that can be determined by solving the system:

$$\gamma_\varepsilon(i) = \gamma_u(i), \quad i = 0, 1, \dots \quad (13)$$

$\gamma_\varepsilon(i)$  and  $\gamma_u(i)$  represent the auto-covariance functions  $\theta(L)\varepsilon_t$  and  $F\Phi^*(L)\theta(L)u_t$ . Dufour and Taamouti proved that the marginal representation of  $x_t$  can be written:

$$\det[\Phi(L)]x_t = F\Phi^*(L)U_t. \quad (14)$$

Combining now the equations (8) and (14) obtain

$$x_t - \varphi_1 x_{t-1} - \varphi_2 x_{t-2} = \varphi_{xy} v_{t-1} - \varphi_{yy} u_{t-1} + u_t, \quad (15)$$

$$\varphi_1 = \varphi_{xx} + \varphi_{yy} \quad \text{and} \quad \varphi_2 = \varphi_{xy}\varphi_{yx} - \varphi_{xx}\varphi_{yy}.$$

Observe now that the right-side of equation (15), denoted by  $\omega_{(t)}$  is the sum of an MA(1) process and a white noise process. By Lemma 5.1,  $\omega_{(t)}$  has an MA(1) representation

$$\omega_t = \bar{\theta}\varepsilon_{t-1} + \varepsilon_t. \quad (16)$$

To determine parameters  $\bar{\theta}$  and  $\sigma_\varepsilon^2$  in terms of the parameters of the unconstrained model, we can solve system (13) for  $v = 0$  and  $v = 1$ ,

$$\text{Var}(\omega_t) = \text{Var}(\varphi_{xy}v_{t-1} - \varphi_{yy}u_{t-1} + u_t) \quad (17)$$

$$\text{Cov}[\omega_t, \omega_{t-1}] = E[(\varphi_{xy}v_{t-1} - \varphi_{yy}u_{t-1} + u_t)(\varphi_{xy}v_{t-2} - \varphi_{yy}u_{t-2} + u_{t-1})] \quad (18)$$

and finally obtain:

$$(1 + \bar{\theta}^2)\sigma_\varepsilon^2 = (1 + \varphi_{yy}^2)\sigma_u^2 + \varphi_{xy}^2\sigma_v^2, \quad \bar{\theta}\sigma_\varepsilon^2 = -\varphi_{yy}\sigma_u^2 \quad (19)$$

because  $\text{Cov}(u,v)=0$ .

Here we have two equations and two unknown parameters  $\bar{\theta}$  and  $\sigma_\varepsilon^2$ . These parameters must satisfy the constraints  $\sigma_\varepsilon^2 > 0$  and  $|\bar{\theta}| < 1$ . To quantify the degree of causality from Y to X at horizon h, we first consider the unconstrained and constrained models of process X. The unconstrained model is

$$x_t = \varphi_{xx}x_{t-1} + \varphi_{xy}y_{t-1} + u_t \quad (20)$$

whereas the constrained model is

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \bar{\theta}\varepsilon_{t-1} + \varepsilon_t \quad (21)$$

Second, we need to calculate the variance-covariance matrices of the unconstrained and constrained forecast errors of  $X(t+h)$ . According to Dufour and Taamouti, we can immediately deduce the causality measure from Y to X denoted by  $CL_{yx}(h)$  at any horizon h using only the unconstrained parameters. This is given by:

$$CL_{yx}(h) = \ln \left[ \frac{\text{Var}[x_{t+h}|x_t]}{\text{Var}[x_{t+h}|x_t, y_t]} \right] \quad (22)$$

where  $\text{Var}[x_{t+h}|x_t]$  represents the variance of the forecast errors of the constrained model given by (21), and  $\text{Var}[x_{t+h}|x_t, y_t]$  represents the variance of the forecast errors of the unconstrained model given by (20). In terms of predictability, this can be viewed as the amount of information brought by the past of Y that can improve the

forecast of  $X(t+h)$ . Following Geweke, this measure can be also interpreted as the proportional reduction in the variance of the forecast error of  $X(t+h)$  obtained by taking into account the past of  $Y$ .

The two variances can be determined as follows (see Dufour and Taamouti):

$$V_1(h) = \text{Var}[x_{t+h}|x_t, y_t] = \sum_{i=0}^{h-1} F \psi_i \Sigma_u \psi_i^T F^t \quad (23)$$

$$V_2(h) = \text{Var}[x_{t+h}|x_t] = \sum_{i=0}^{h-1} F \bar{\psi}_i \Sigma_\varepsilon \bar{\psi}_i^T F^t \quad (24)$$

where  $\bar{\psi}_i$  can be determined in the same way as  $\psi_i$  from equation (10). Obviously this measure causality is non-negative and is zero, only if there is no causal relationship between the two variables. The causality is higher, the higher is the measure. In terms of predictability, this could be interpreted by the amount of information brought by the variable  $y$  in predicting variable  $x$ .

For the case of a VAR model (1) we can analytically determine causality measure at a horizon equal to  $h$ , using only unrestricted model parameters. For example, the measure causality from  $Y$  to  $X$ , from a horizon equal to one and two, are given by very simple relations (see Dufour and Taamouti, working paper version).

### 3. Causality GDP - Investment in education

The data necessary to this study refers to gross domestic product and investment for education, both in terms of per-capita quantities, in 2005 constant prices and were obtained for the three countries, namely France, Germany and Romania, from database of the World Bank. As expected, these series are not stationary, an assertion confirmed with Dickey-Fuller test. Consequently, we used the transformed time series, i.e. in differences of logarithm to eliminate in this way potential serial correlation. According to the same test, the transformed time series are all stationary. Then, using a conventional model to estimate the parameters of the model VAR (1) we obtained the following results for the three countries:

Romania:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0.0147 \\ -0.0019 \end{bmatrix} + \begin{bmatrix} 0.445348 & 0.118334 \\ 0.990355 & 0.122812 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix}$$

Germany:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0.0100 \\ 0.0144 \end{bmatrix} + \begin{bmatrix} -0.100967 & 0.177284 \\ 0.050164 & 0.487530 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix}$$

France:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0.0026 \\ 0.0181 \end{bmatrix} + \begin{bmatrix} 0.356579 & 0.198782 \\ -0.143363 & 0.122613 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix}$$

The roots of the lag polynomials are:  $(-10.60; 1.51)$ ,  $(-8.64; 1.99)$  and respectively  $(3.32 + 1.69i; 3.32-1.69i)$ . As we can see, all these roots are superior to one, in absolute value, and consequently I can claim that the three VAR (1) models are stationary. Using now the relations (23), (24) and the residual variances of the three models estimated above, we can determine a measure of causality for a horizon equal to one and two.

- I. We will analyze in a first stage the causality relation between investment in education and gross domestic product. Below is presented in a detailed manner the computational procedure, only for the Romanian economy.
  1. We first determine the prediction of error variance of variable  $x$ , at a horizon equal to one and then equal to two, using both its own past and that of the variable  $y$ . Using equation (23) for  $h = 1$  and  $h = 2$  we have:

$$V_1(1) = [1 \ 0] \begin{bmatrix} \varphi_{xx} & \varphi_{xy} \\ \varphi_{yx} & \varphi_{yy} \end{bmatrix}^0 \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{vu} & \sigma_v^2 \end{bmatrix} \begin{bmatrix} \varphi_{xx} & \varphi_{yx} \\ \varphi_{xy} & \varphi_{yy} \end{bmatrix}^0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0.00070049$$

$$V_1(2) = V_1(1) + [1 \ 0] \begin{bmatrix} \varphi_{xx} & \varphi_{xy} \\ \varphi_{yx} & \varphi_{yy} \end{bmatrix} \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{vu} & \sigma_v^2 \end{bmatrix} \begin{bmatrix} \varphi_{xx} & \varphi_{yx} \\ \varphi_{xy} & \varphi_{yy} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0.00087966.$$

2. We will now determine the prediction error variance of the variable x using only its past.  $\bar{\theta}$  and  $\sigma_\varepsilon^2$  are given by the relations (19).  $\bar{\theta} = -0.11605$  and  $\sigma_\varepsilon^2 = 0.00074131$ . We now need to determine the structure of the model restricted to a horizon of  $t + 1$  and  $t + 2$  then. Applying equation (21) we obtain

$$x_{t+1} = \varphi_1 x_t + \varphi_2 x_{t-1} + \bar{\theta} \varepsilon_t + \varepsilon_{t+1},$$

$$x_{t+2} = (\varphi_1^2 + \varphi_2) x_t + \varphi_1 \varphi_2 x_{t-1} + \varphi_1 \bar{\theta} \varepsilon_t + (\varphi_1 + \bar{\theta}) \varepsilon_{t+1} + \varepsilon_{t+2}$$

from where we it follows:

$$V_2(1) = \sigma_\varepsilon^2 = 0.00074131 \text{ and } V_2(2) = [(\varphi_1 + \bar{\theta})^2 + 1] \sigma_\varepsilon^2 = 0.00089284.$$

Substituting these results into the relation (22), we get:

$$CL_{xy}(1) = 0.0566 \text{ and } CL_{xy}(2) = 0.0148.$$

The complete results concerning the causality between investment in education and gross domestic product are presented in the table below.

Country	V <sub>1</sub> (1) (10 <sup>-3</sup> )	V <sub>1</sub> (2) (10 <sup>-3</sup> )	V <sub>2</sub> (1) (10 <sup>-3</sup> )	V <sub>2</sub> (2) (10 <sup>-3</sup> )	CL(1)	CL(2)
Romania	0.7005	0.8790	0.7410	0.8930	0.0566	0.0148
Germany	0.5006	0.5371	0.5409	0.5431	0.0773	0.0111
France	0.1976	0.2707	0.2461	0.2818	0.2196	0.0403

- II. Proceeding in the same way as above, we can now examine the causal relationship between GDP and investment in education. In this case, the vector F of the equation (14) is of the form  $F = [0 \ 1]$  and the equation (15) is written as follows:

$$y_t - \varphi_1 y_{t-1} - \varphi_2 y_{t-2} = \varphi_{yx} u_{t-1} - \varphi_{xx} v_{t-1} + v_t, \quad (15')$$

that finally gives:

$$(1 + \bar{\theta}^2) \sigma_\varepsilon^2 = (1 + \varphi_{xx}^2) \sigma_v^2 + \varphi_{yx}^2 \sigma_u^2, \quad \bar{\theta} \sigma_\varepsilon^2 = -\varphi_{xx} \sigma_v^2 \quad (19')$$

1. We first determine the prediction of error variance of variable y, at a horizon equal to one and then equal to two, using both its own past and that of the variable x. Using the same equation (23), but this time with  $F = [0 \ 1]$ , for  $h = 1$  and  $h = 2$  we have:

$$V_1(1) = [0 \ 1] \begin{bmatrix} \varphi_{xx} & \varphi_{xy} \\ \varphi_{yx} & \varphi_{yy} \end{bmatrix}^0 \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{vu} & \sigma_v^2 \end{bmatrix} \begin{bmatrix} \varphi_{xx} & \varphi_{yx} \\ \varphi_{xy} & \varphi_{yy} \end{bmatrix}^0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0.00287299$$

$$V_1(2) = V_1(1) + [0 \ 1] \begin{bmatrix} \varphi_{xx} & \varphi_{xy} \\ \varphi_{yx} & \varphi_{yy} \end{bmatrix} \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{vu} & \sigma_v^2 \end{bmatrix} \begin{bmatrix} \varphi_{xx} & \varphi_{yx} \\ \varphi_{xy} & \varphi_{yy} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0.00360337.$$

2. We will now determine the prediction error variance of the variable y using only its past.  $\bar{\theta}$  and  $\sigma_\varepsilon^2$  are given by the relations (19').  $\bar{\theta} = -0.34715$  and  $\sigma_\varepsilon^2 = 0.00368568$ . We now need to determine the structure of the model restricted to a horizon of  $t + 1$  and  $t + 2$  then. Applying equation (21), written this time for y, we obtain:

$$y_{t+1} = \varphi_1 y_t + \varphi_2 y_{t-1} + \bar{\theta} \varepsilon_t + \varepsilon_{t+1},$$

$$y_{t+2} = (\varphi_1^2 + \varphi_2)y_t + \varphi_1\varphi_2y_{t-1} + \varphi_1\bar{\theta}\varepsilon_t + (\varphi_1 + \bar{\theta})\varepsilon_{t+1} + \varepsilon_{t+2}$$

from where it follows that:

$$V_2(1) = \sigma_\varepsilon^2 = 0.00368568 \text{ and } V_2(2) = [(\varphi_1 + \bar{\theta})^2 + 1]\sigma_\varepsilon^2 = 0.00386571.$$

Substituting these results into the relation (22), modified for the causality  $x \rightarrow y$ , we get:

$$CL_{yx}(1) = 0.2491 \text{ and } CL_{yx}(2) = 0.0703.$$

The complete results concerning the causality between gross domestic product and investment in education are presented in the table below.

Country	$V_1(1)$ ( $10^{-3}$ )	$V_1(2)$ ( $10^{-3}$ )	$V_2(1)$ ( $10^{-3}$ )	$V_2(2)$ ( $10^{-3}$ )	CL(1)	CL(2)
Romania	2.873	3.603	3.686	3.866	0.2491	0.0703
Germany	0.998	1.237	0.999	1.237	0.00127	0.00015
France	1.213	1.235	1.218	1.236	0.00383	0.00086

#### 4. Conclusions and consequences

Analyzing the above results we can conclude that in all three cases the causality measure is positive and persistent, both in terms of the effect of investment in education on gross domestic product and in terms of the reverse effect. In terms of numerical dimensions, they confirm the importance given to education in the three countries. Although for the French economy, the level of causality seems to be a little bit excessive compared with that of Germany, although it reflects undoubtedly a trend - France is among the European countries which allocates considerable resources in the education. Furthermore, it is the country where the private education system is almost non-existent.

Romanian economy presents the lowest degree of causality, which can only confirm the extremely low resources allocated to education. However, what can be seen from the results is that with increasing gross domestic product, investment in education began to increase substantially, claim justified by the coefficient CL (1) of the final table.

What can be seen from the above results is that in Germany and France, the causal effect of GDP on investment in education is significantly lower than that of the causal effect of investment in education on gross domestic product, while in Romania, it is exactly the opposite. This can be explained by the fact that the two countries have reached a level of relative stability in the resources allocated to education. In Romania, however, these results confirm that further efforts are still required to finance education system.

Obviously, the analysis can be questioned here by the limited number of statistical data taken into account - the period 1991-2013, but this was the only available period, particularly for the Rumanian economy.

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