Individual contributions to portfolio risk: risk decomposition for the BET-FI index

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Abstract

The paper applies Euler formula for decomposing the standard deviation and the Expected Shortfall for the BET-FI equity index.

Risk attribution allows the decomposition of the total risk of the portfolio in individual risk units. In this way we can compute the contribution of each company to the overall standard deviation/Expected Shortfall of the portfolio.

Keywords: *risk attribution, marginal contributions, Expected Shortfall*

JEL: *C1, G11*

1. Introduction

A portfolio contains a large number of positions on different financial instruments ranging from stock, bonds to derivatives instruments. In order to model the risk and return of the portfolio it is necessary to map the portfolio to its risk factors. The risk factors for a linear portfolio may include the prices of general market indices, foreign exchange rates or zero coupon market interest rates of different maturities to which the portfolio is exposed.

The risk factor sensitivities of an asset or portfolio measure the change in price when a factor risk changes while holding constant the other factors. In a stock portfolio the risk factor sensitivities are called betas (factor betas). In a linear portfolio, such as a stock portfolio, the mapping of the risk factors is carried out with factor models.

Risk is expressed as a sum of the contribution from each factor contributing to the overall risk evaluation. If a portfolio of securities may be mapped to a set of factors then the factors should explain most of the variation in the portfolio. APT factor models explain the mapping by segregating between systematic and idiosyncratic components. By using Euler formula it is possible to decompose the contribution of each factor and to assess the specific contribution of each factor.

Yamai (2002) and Hallerbach (2003) showed that Value at Risk can be decomposed in several components: marginal VaR, component VaR and incremental VaR assuming Gaussian distribution. The marginal VaR is the marginal contribution of the individual portfolio component to the diversified portfolio VaR, component VaR is the proportion of the diversified portfolio VaR that is attributed to the individual components and incremental VaR is the effect on the VaR of the portfolio by adding a new financial instrument.

Zhang and Rachev (2004) criticized the beta coefficient from the Capital Asset Pricing Model (CAPM) since it is neither translation-invariant nor monotonic, properties that any coherent risk measure should display. The authors define "risk attribution' as "a process of attributing the return of a portfolio to different factors according to active investment decisions". They show that by using Euler's formula it is possible to identify the main sources of risk in a portfolio.

Scherer (2005) implemented risk budgeting with multiple benchmarks and rival risk regimes for accommodating the different objectives demanded by investors.

Darolles and Gouriéroux (2012) applied the risk contribution restrictions on a portfolio of futures on commodities and compared the performance of the associated portfolios in terms of risk contributions, performance and budget allocations.

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2. Factor Models

A factor model allows the analysis of the returns of a portfolio and computation of the portfolio risk. Factor models are based on univariate or multivariate linear regression. Capital Asset Pricing Model (CAPM) is a Single Factor Model (Single Index Model) which assumes a linear dependence between the expected excess returns of a single asset and the expected excess return on the market portfolio and allows to investigate the risk and return characteristic of an asset relative to the market index.

The Single Index Factor Model uses a broad market index (F_M) as a proxy for the market portfolio which is unknown. Roll (1977) showed that the market portfolio is not observable. Fama and French (1992) showed that beta and long-run average return are not correlated.

$$E(R_i) = \alpha + \beta_i E(F_M) \quad (1)$$

$$\beta_i = \frac{Cov(R_i, F_M)}{\sigma_F^2} \quad (2)$$

where F_M represents the ordinary return (not the excess returns) on the market portfolio.

$$R_i = \alpha + \beta_i F_{M,t} + \varepsilon_{it}, \varepsilon_{it} \sim i.i.d(0, \sigma_i^2) \quad (3)$$

where β_i is the risk factor sensitivity of the asset *i*, $\beta_i \sigma_F$ is the systematic volatility of the asset *i*, σ_i is the specific volatility of the asset *i* and σ_F is the volatility of the equity index.

In a Single Factor Model the total risk is decomposed in systematic risk and specific risk. The volatility of the portfolio return can be decomposed in three risk sources: 1) sensitivity to the market factor beta 2) volatility of the market factor and 3) specific risk.

Beta is a measure of risk in a portfolio since the weighted sum of individual betas equals the portfolio beta.

$$\beta_P = \sum_{i=1}^{N} w_i \beta_i \quad (4)$$

When considering a portfolio with *k* factors, the linear Multifactor Model may be written as

$$R_t = \alpha + \beta_{i1} * F_{1t} + \cdots \beta_{ik} * F_{kt} + \varepsilon_{it} \quad (5)$$

$$\begin{aligned} R_t &= \alpha + \beta_i * F_t + \varepsilon_{it}, F_t \sim (\mu_F, \Sigma_F \\ \varepsilon_{it} &= (0, \sigma_{\varepsilon,i}^2) \end{aligned} \quad (6)$$

Since the asset returns are typically non-normal, not i.i.d, there are several distributions that may fit the data better than the Gaussian distribution. Among the distributions usually used in practice for fitting financial returns are Student's-*t*, skewed Student-*t*, GED, generalized Pareto, etc. Since in practice the portfolios may include large number of assets sometimes in small samples and with missing data, the multivariate modelling involved by the portfolio analyses are difficult and may require a non-parametric fitting of the multivariate distribution.

We may use statistical factor model for quantifying the portfolio exposures to risk factors. Our drive is to map the risk factors in equity portfolio and to decompose their contribution to the portfolio risk.

In order to quantify the portfolio risk, we use the following risk measure: standard deviation (SD) and Expected Shortfall also known as conditional Value at Risk (cVaR) or Expected Tail Loss (ETL).

The risk measures (RM) are defined as:

Active risk (SD) is derived from the variance of the portfolio

$$SD = \sqrt{\beta_i' \Sigma_F \beta_j + \sigma_\varepsilon^2} \quad (7)$$

Value at Risk (VaR)

$$VaR_\alpha = F^{-1}(\alpha), where\ F\ is\ the\ c.df.of\ R_t \quad (8)$$

Expected Shortfall (ES)

$$ES_\alpha = E[R_t | R_t \leq VaR_\alpha] \quad (9)$$

The risk decomposition is performed by highlighting the contribution of each risk factor and the contribution of constituent assets to portfolio risk. Euler formula shows that if R(w) is the scalar risk measure associated with allocation and if the risk measure is homogeneous of degree 1 that is $RM(\lambda w) = \lambda RM(w)$ then by differentiating the homogeneity condition with respect to λ we get:

$$\sum_{i=1}^{N} w_i \frac{\partial RM(\lambda w)}{\partial w_i} = RM(w) \quad (10)$$

and deduce the Euler formula by setting $\lambda = 1$

Reverting to the factor model, we get additive decomposition by using Euler theorem

$$RM_p(w) = \sum_{j=1}^{k+1} w_i \frac{\partial RM(w)}{\partial w_i} \; where\ RM\ is\ SD, VaR_\alpha, ES_\alpha \quad (11)$$

where *w* are the portfolio weights and RM denotes a portfolio risk measure that is a homogenous function of degree one in the portfolio weight vector. RM may be standard deviation, Value at Risk or Expected Shortfall.

Since portfolio volatility is a linear homogenous function of portfolio weights, the portfolio volatility may be re-written as the weighted sum of marginal risk contribution.

By taking into account the weights of the asset in the composition of the portfolio, the contributions of the individual asset to the portfolio risk are quantified as:

MCRs: Asset *i* marginal contribution to portfolio risk:

$$\frac{\partial RM(w)}{\partial w_i} \quad (12)$$

CRs: Asset *i* contribution to portfolio risk:

$$w_i \frac{\partial RM(w)}{\partial w_i} \quad (13)$$

The asset *i* contribution to risk are weighted marginal contributions.

PCRs: Asset *i* percent contribution to portfolio risk:

$$w_i \frac{\partial RM(w)}{\partial w_i} \Big/ RM(w) \quad (14)$$

The asset percent contributions (PCR) to portfolio risk measure are the contributions to risk divided by the risk measure (RM).

Each component contribution to risk of asset *i* represents the amount of risk contributed to the total risk by investing a certain weight (w_i) in asset *i*. The sum of all contributions equals the total risk. By rescaling the contribution to risk of asset I, we get the percentage of the total risk which is contributed by asset *i*. The sum of asset *i* percent contribution to portfolio risk sum up to 1 (100%).

The marginal contribution to risk of asset *i* represents the marginal impact in the total portfolio risk which comes from a small change in the weight attributed to asset *i*. If the sign of the marginal risk is negative, then by increasing the position size of the asset we increase the total portfolio risk and vice versa.

An incremental change in the allocation of asset *i* is offset by a corresponding change in the allocation of asset *j* such as $\Delta w_i = -\Delta w_j$.

Therefore the change in portfolio volatility is about:

$$\Delta\sigma_{porfolio} = (MCR_i^{\sigma} - MCR_j^{\sigma})\Delta w_i \quad (15)$$

Scaillet (2002) and Meucci (2007) showed that we if assume a multivariate Gaussian distribution for the financial returns, then the partial derivative that give the assets' contribution to risk can be computed analytically while in non-normal markets their joint distribution can be represented through Monte Carlo simulations.

3. Data

Our dataset includes the component companies of the BET-FI index: Fondul Proprietatea (symbol:FP), SIF1, SIF2, SIF3, SIF4, SIF5. The closing prices for BET-FI index and Fondul Proprietatea (symbol:FP), SIF1, SIF2, SIF3, SIF4, SIF5 were extracted from Bucharest Stock Exchange website from January 2014 to April 2015. Daily returns were calculated from the closing prices according to the formula $R_t = \ln(P_t/P_{t-1})$ where P_t is the daily closing price of the index.

Since the weights of the component companies were often changed during the sample time period although with small differences, we have constructed a proxy portfolio for the BET-FI index using the average weights displayed in Table 1. All the calculations were carried out on the proxy portfolio.

Table 1. BET-FI component' weights

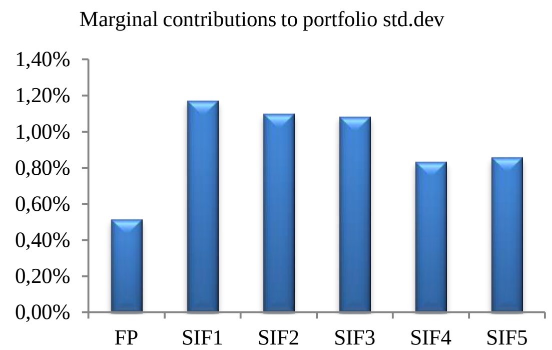
	FP	SIF5	SIF4	SIF2	SIF1	SIF3
14.03.2014	0.2989	0.2145	0.1361	0.1241	0.1182	0.1082
11.06.2014	0.3127	0.1995	0.1257	0.1252	0.119	0.117
13.06.2014	0.2996	0.2016	0.1297	0.129	0.1212	0.1189
01.08.2014	0.2998	0.1995	0.1299	0.1257	0.123	0.1221
12.09.2014	0.2996	0.1947	0.1418	0.1246	0.1236	0.1157
12.12.2014	0.2994	0.1842	0.1501	0.133	0.1309	0.1024
13.03.2015	0.2989	0.1789	0.1492	0.1388	0.1276	0.1065
Average	0.30127	0.19612	0.1375	0.128629	0.12335	0.11297

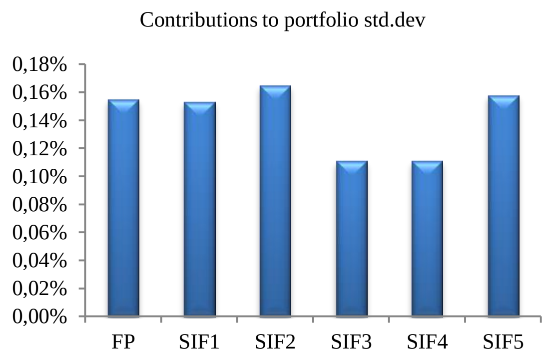
4. Results

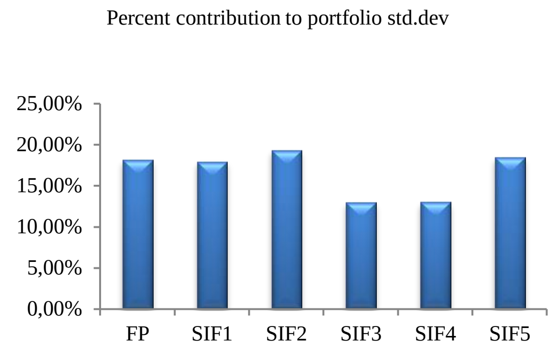
Since the marginal contribution to risk of FP is lowest among all the other companies, any incremental change in the weight allocated to FP will decrease the overall portfolio volatility. On the other hand, since the marginal contributions of the other five companies are almost the same, any change in them will increase the volatility in a smaller degree.

Given the high weight of the FP the percent contribution of all six companies to the portfolio volatility is close, ranging from 13% to 18%. SIF1 has the highest marginal contribution although it has a small weight.

Figure 1. Standard deviation decomposition



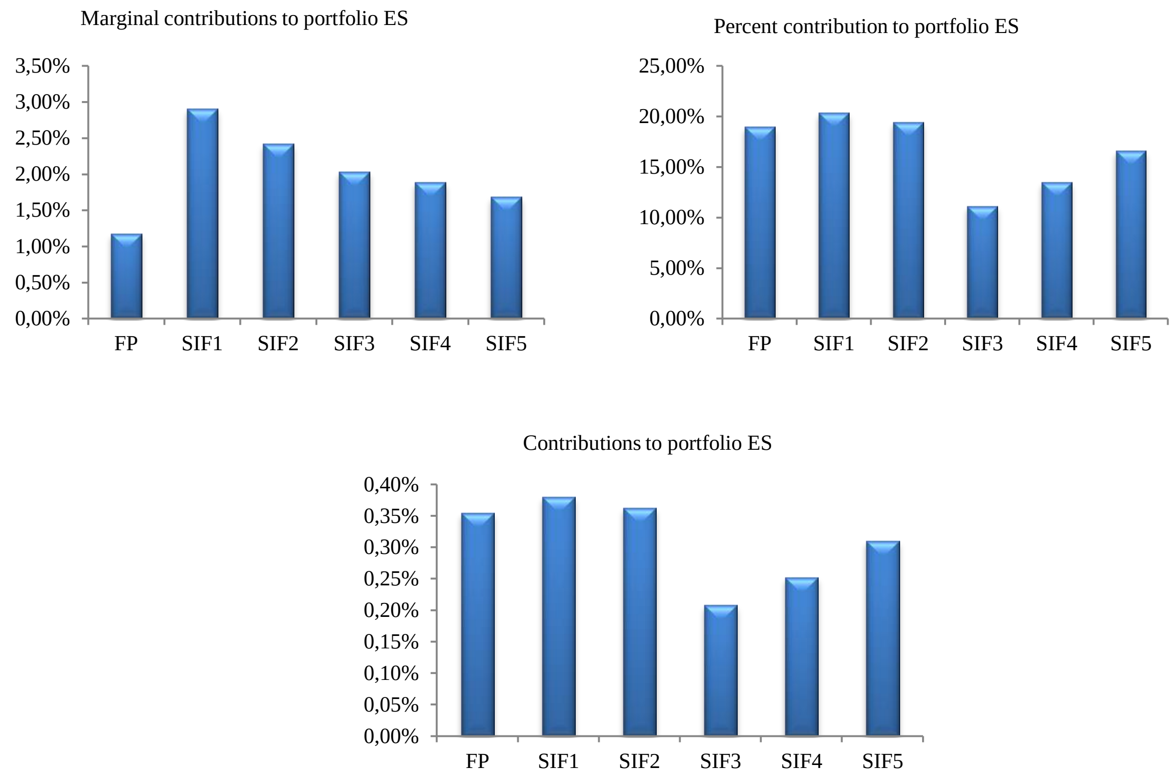




Source: Author's calculations

The VaR of BET-Fi portfolio at 5% is 1.15% and the Expected Shortfall at 5% is 1.86%. Using the Euler formula in the same way as for the portfolio standard deviation, we can compute the MCR, CR and PCR of the contribution of the constituent companies of the BET-FI index to the Expected Shortfall. The results show that in the case of PCR, FP, SIF1 and SIF2 contribute with about the same percent, around 20%. SIF3 and SIF4 have the lowest contribution with 11.15% and 13.5% and SIF4 contributes with 16.6%. The marginal contributions to portfolio ES is similar to the marginal contributions to standard deviation.

Figure 2. Expected Shortfall decomposition



Source: Author's calculations

5. Conclusion

Risk attribution is a method of decomposing the portfolio risk and attributing the return of a portfolio to its risk factors. Thus it is possible to calculate the assets' return contribution to the portfolio standard deviation/Expected Shortfall.

If the chosen risk measure is a homogenous function of degree one in the portfolio weight vector, then we can apply the Euler formula to decompose it into individual contributions.

We have applied Euler formula in order to decompose the standard deviation and the Expected Shortfall of the BET-FI equity index into individual risk contribution.

Our dataset included the component companies of the BET-FI index: Fondul Proprietatea (symbol:FP), SIF1, SIF2, SIF3, SIF4, SIF5. We have constructed a proxy portfolio for the BET-FI index for taking by taking into account the average weight of the companies included in the index portfolio.

The contributions of the individual asset to the portfolio risk are quantified as asset *i* marginal contribution to portfolio risk (MCR), asset *i* contribution to portfolio risk (CR), asset *i* percent contribution to portfolio risk (PCR).

The results showed that in average the marginal contribution to risk of FP was the lowest among all the other companies, meaning that an incremental change in the weight allocated to FP will decrease the volatility of the BET-FI index. The marginal contributions of the other five companies (SIFs) were similar implying that any change in their allocation will increase the portfolio volatility in a smaller degree and smaller degree.

The percent contribution of all six companies to the portfolio volatility is close, ranging from 13% to 18%. SIF1 has the highest marginal contribution although it has a small weight.

We computed the MCR, CR and PCR of the contribution of the constituent companies of the BET-FI index to the Expected Shortfall. FP, SIF1 and SIF2 contributed with about the same percent (20%), SIF3 and SIF4 had the lowest contribution (11.15% and 13.5%) and SIF4 contributed with 16.6%.

By using the risk budgeting framework it is possible to decompose the risk measure(s) calculated on a portfolio and compare the performance of any portfolio in terms of risk contributions, performance and budget allocations.

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References

[1] Darolles, S., Gouriéroux, C., & Jay, E, “Portfolio allocation with budget and risk contributions restrictions” CREST DP, 2012.

[2] Hallerback, W.G.,*”Decomposing Portfolio Value-at-Risk: A General Analysis”,* The Journal of Risk, 5(2), 1-18, 2003

[3] Meucci, A., “*Risk contributions from generic user-defined factors”,* RISK-LONDON-RISK MAGAZINE LIMITED-, 20(6), 84, 2007.

[4] Roll, R., “*A Critique of the Asset Pricing Theory.s Tests: Part I”,* Journal of Financial Economics, 4, 129-176,1977.

[5] Scaillet, O., ”Nonparametric estimation and sensitivity analysis of expected shortfall”,. Mathematical Finance, vol. 14, 74-86, 2002.

[6] Scherer, B., Martin, R. D., “*Modern Portfolio Optimization with NuOPT™, S-PLUS®, and S+ Bayes™*”, Springer Science & Business Media, 2005.

[7] Yamai, Y., & Yoshiba, T., “Comparative analyses of expected shortfall and value-at-risk: their estimation error, decomposition, and optimization”, Monetary and economic studies, 20(1), 87-121, 2002.

[8] Zhang, Y., Rachev, S., *“Risk Attribution and Portfolio Performance Measurement-An Overview*”, University of Karlsruhe, D-76128 Karlsruhe, Germany and Department of Statistics and Applied Probability University of California, Santa Barbara, CA93106, USA, 2004.