Bayesian modelling of real GDP rate in Romania

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Abstract

The main objective of this study is to model and predict the real GDP rate using Bayesian approach. A Bayesian VAR (BVAR), a Bayesian linear model and switching regime Bayesian models were employed for the real GDP rate, inflation rate and interest rate. From the set of variables that were connected to real GDP, for identifying the most relevant ones using the data for Romanian economy, we applied the selection algorithm based on stochastic search. Weight of revenues in GDP, weight of budgetary deficit in GDP, investment rate and inflation rate are the most correlated variables with the real GDP rate. The averages of posterior coefficients of models were used to make forecasts. For Romania on the horizon 2011-2014, the unrestricted switching regime models generated the most accurate forecasts.

Keywords: *Bayesian model, forecasts, GDP rate, switching regime*

1. Introduction

The main aim of this study is to propose various types of Bayesian models to predict the real GDP in Romania. Moreover, a Bayesian algorithm was applied to select the variables that explain better the evolution of real GDP rate in Romania. The advantages of Bayesian approach are essential for the case of Romanian economy when the lack of long time series is a serious problem. Bayesian methods have a general character that does not require special regularity conditions, concepts like confidence interval and assumptions' testing. The Bayesian approach conducts us to the evaluation of the VAR and linear regression models by using Bayesian principles. These types of models were employed in this study for annual data. Moreover, the proposed econometric models were used to construct some predictions during 2011-2014.

After this Introduction, a literature review is made, being followed by the estimation of Bayesian models that are used in making forecasts. The results indicated a good accuracy of real GDP forecasts during 2011-2014.

2. Literature review

In Romania a BVAR model for quarterly GDP was built by a researcher who made the comparison with unrestricted VAR model, a OLS regression and random walk in terms of forecasts accuracy. The results put into evidence a slow recovery of the Romanian economy, for the next quarters (2009Q4-2010Q4) the Bayesian model predicting a negative gap [1].

The real GDP growth rate depends on investment rate, following the relationship [2]:

$$\mathrm{rGDP_t} = \mathrm{a} + \mathrm{b} \cdot \mathrm{rinv_t} + \mathrm{eps_t} \quad (1)$$

$rGDP_t$- real GDP growth rate at time t

$rinv_t$- investment rate at time t

eps_t- error term

a,b- parameters to be estimated

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Some authors more multiple regression models for explaining the GDP growth rate [3]:

$$\mathrm{rGDP_t} = \alpha_0 + \alpha_1 \cdot \mathrm{income\_tax_t} + \alpha_2 \cdot \mathrm{expenses\_weight_t} + \alpha_3 \cdot \mathrm{deficit_t} + \varepsilon_{1t} \quad (2)$$

$$\mathrm{rGDP_t} = \alpha_0 + \alpha_1 \cdot \mathrm{income\_tax_t} + \alpha_2 \cdot \mathrm{expenses\_weight_t} + \alpha_3 \cdot \mathrm{deficit_t} + \alpha_4 \cdot \mathrm{BCF_t} + \alpha_5 \cdot \mathrm{rpop_t} + \varepsilon_{2t} \quad (3)$$

$$\mathrm{rGDP_t} = \alpha_0 + \alpha_1 \cdot \mathrm{income\_tax_t} + \alpha_2 \cdot \mathrm{expenses\_weight_t} + \alpha_3 \cdot \mathrm{other\_revenues_t} + \alpha_4 \cdot \mathrm{GCF_t} + \alpha_5 \cdot \mathrm{rpop_t} + \varepsilon_{3t} \quad (4)$$

$rGDP_t$- real GDP growth rate

$income\_tax_t$- weight of tax in GDP

$expenses\_weight_t$- weight of total expenses in GDP

$deficit_t$- weight of deficit or surplus in GDP

$other\_revenues_t$- weight in GDP of net revenues from other sources except for dues

GFC_t- weight in GDP of gross fixed capital formation

$rpop_t -$ population growth rate

$\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}$- errors terms

Marcellino, Porqueddu and Venditti predicted the GDP in euro zone using the Bayesian indicator of growth. EURO-BIG is actually a mixed frequency data small scale dynamic factor model with stochastic volatility [4].

Recently, a Bayesian variant of global vector autoregressive model (B-GVAR) was proposed. The predictive performance of B-GVAR models was compared for variables like inflation rate, real GDP, real exchange rate and interest rates. By considering the international linkages the inflation, real GDP and the real exchange rate predictions are improved [5].

In order to predict the GDP growth in euro zone, some authors employed bridge models which were applied for aggregate GDP. The national bridge model predicted better the GDP rate than univariate and multivariate models that were used as benchmark. The aggregation of national predictions generated an improvement in accuracy [6]. It was proved that an improvement in GDP forecasts accuracy was brought by the use of monthly data regarding the current activity more than the use of financial variables from surveys. A large number of time series with monthly and quarterly frequency in real-time was used to predict the output growth. The performance of the factor model forecasts was assessed by comparison with the GDP of Germany. The importance of predictions revisions was analyzed in detail [8].

The real GDP rate was nonlinearly correlated with oil prices, this relationship being used to predict the real GDP growth in USA. The symmetric nonlinear processes seemed to provide more accurate forecasts than the asymmetric models [9]. The logistic-growth equations were used to predict the real GDP per capita and long-run inflation [10].

3. Modelling real GDP rate in Romania

For constructing the econometric models that explains the evolution of GDP growth rate, the data series for the following variables have been used: GDP growth rate, the weight in GDP of revenue tax, budget expenses, budget revenues, budget deficit, gross fixed capital formation, population growth rate, inflation rate, unemployment rate, interest rate, investment rate. The sources of data are Eurostat and National Institute of Statistics and the period is 1991-2013.

A Bayesian VAR (BVAR) of order 2 was employed for the real GDP rate, inflation rate and interest rate. In Appendix 1 the posterior coefficients, the posterior covariance matrix and constant terms were presented. For making predictions of real GDP growth, we used the average of posterior coefficients. 10 000 replications were saved for this application, the total number of iterations being 50 000. The form of BVAR model is the following one:

$$Y(t) = c + Phi(1)*Y(t-1) + ... + Phi(p)*Y(t-p) + ut \quad (5)$$
$$ut \sim MVN(0,Sigma), Y(t) \text{ inclused d variables}$$

The Gibbs sampler algorithm with prior values as in Lindley and Smith (1972) was applied:

Phi ~ N(mu,V), Sigma ~ W(Omega,df)

The results consist in:

- Phi_draws = posterior values of phi(1);
- Sigma_draws = posterior values for covariance matrix;
- constant_draws = posterir values for constant.

Posterior conditional distribution Sigma follows an inverse Wishart distribution.

With Lindley and Smith (1972) proper priors, draws from posterior conditionals of Beta and Sigma2 are obtained successively. When the priors are flat, the posterior means of Beta and Sigma2 are similar to the OLS estimator.10 000 replications were saved from this application, the total number of iterations being 50 000.

The GDP growth rate is explained using the weight of deficit in GDP, the weight of tax in GDP and the weight of gross capital formation in GDP using linear Bayesian models. In the Appendix 2 posterior mean and posterior standard deviations were displayed for coefficients and for the errors variance.

The switch of regime breaks the regression into two regression models with different slope and disturbances variance. The regime switch time is unknown and has a uniform prior. Then the posterior is proportional to the likelihood in which switch occurs at a given time. Priors and posteriors of other parameters follow the standard linear regression model.

For the real GDP growth a Bayesian switching regime model was proposed. In Appendix 3 posterior mean and posterior standard deviations were displayed for coefficients and for the errors variance before and after the regime change. Two variants of the model were proposed: unrestricted regime and connected regime.

From the set of variables that were connected to real GDP, for identifying the most relevant ones using the data for Romanian economy, we applied the selection algorithm based on stochastic search, being proposed by George and McCulloch (1997).

The form of the model is:

$$Yi = Xi * Betai + ui \text{ , where } ui \sim N(0,s^2) \quad (6)$$

where Betai | taui ~taui * N(0,V1) + (1-taui) * N(0, V2) , V1 > V2

taui = 1 suggests a variable is chosen, while taui = 0 implies Betai is close to zero and can be excluded

Gibbs sampler with hierarchical proper priors was used.

At level one: s^2 ~ IG(a,b), Betai | taui ~ taui * N(0,V1) + (1-taui) * N(0, V2)

Level two supposes: taui | pi ~ Bernoulli (pi)

Level three imples: pi ~ Beta (a', b')

Hyperparameters are specified below. Conditional posteriors of Betai, s^2, pi have conjugate forms, while conditional posterior of taui is updated by the Bayes formula.

Y = dependent variable (n * 1 vector)

X = regressors (n * k matrix)

ndraws = number of draws in MCMC

burn_in = number of burn-in draws in MCMC

tol = the critical probability to accept of variable add_constant = whether to add a constant to X (default = 0)

The results are:

Beta= posterior draws of coefficients corresponding to the k regressors

Sigma2 = posterior draws of variance of disturbances

Tau = posterior draws of the variable inclusion indicators

Beta_refine = posterior draws of coefficients of the refined regression model

Sigma2_refine = posterior draws of disturbances variance of the refined regression model

The prior of regression coefficients have a Gaussian mixture, one with large variance and one with small variance. If a coefficient resides on the latter, it is an indication of exclusion since the coefficient is close to zero. The acceptance critical probability is 0.3.

In the following table the final results of the algorithm application were presented, the details being in Appendix 4.

Table 1. The results of the application of Bayesian algorithm for selecting the determinant factors for real GDP rate

Current no.	Variable	Decision(excluded/included)
1	Rate of population growth	Excluded
2	Weight of revenues in GDP	**Included**
3	Weight of expenses in GDP	Excluded
4	Weight of budgetary deficit in GDP	**Included**
5	Weight of gross capital formation in GDP	Excluded
6	Weight of tax on income in GDP	Excluded
7	Investment rate	**Included**
8	Interest rate	Excluded
9	Inflation rate	**Included**
10	Unemployment rate	Excluded

Source: own computations

From the table, we can see that only four variables were considered in the final model: weight of revenues in GDP, weight of budgetary deficit in GDP, investment rate and inflation rate. The predictions based on the proposed Bayesian models were displayed in the following table.

Table 2. Forecasts of real GDP growth (%) using Bayesian models (horizon: 2011-2014)

Year	Bayesian linear regression model	BVAR(2) model	Bayesian model with unrestricted switching regime	Bayesian model with connected switching regime	Registered values
2011	2.87	2.83	2.75	3.4	2.3
22012	4.04	1.07	2.7	4.8	0.6
22013	4.79	1.46	2.8	5.8	3.5
22014		0.48			

Source: own computations

In the category of Bayesian model, we observed that for 2011 and 2013, the Bayesian model with unrestricted switching regime generated the closest value of the prediction for these years. For 2012, the BVAR model determined the most accurate prediction, while for 2014 the unrestricted

Conclusion

The Bayesian approach is useful for modeling and predicting macroeconomic variables. In the context of economic crisis, many researchers raised questions regarding the ability of the economists to better predict the macroeconomic indicators. The failure of usual econometric model that did not anticipated the economic crisis made the economists to consider better solutions like Bayesian models.

For Romania, during 2011-2014, the unrestricted switching regime models generated the most accurate forecasts.

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APPENDIX 1

BVAR model for real GDP growth, inflation rate and interest rate

Constant

1.8778	5.6555	-2.5507

Posterior Phi1 coefficients

0.21	0.02	-0.21
-0.35	0.62	0.27
-0.10	-0.01	0.99

Posterior Phi2 coefficients

0.09	0.01	0.17
-1.75	-0.11	0.07
0.50	0.08	-0.12

Posterior covariance matrix of the VAR system

9.93	-38.69	-14.28
-38.69	1572.85	269.61
-14.28	269.61	113.74

APPENDIX 2

Bayesian linear regression model

The dependent variable : real GDP rate

The regressors: budgetary deficit, tax, gross capital formation.

A constant is added to regressors.

'Coeff.'	'Post. mean'	'Post. std'
'C(0)'	[-11.1979]	[5.2553]
'C(1)'	[-0.9120]	[0.4991]
'C(2)'	[0.0018]	[0.0374]
'C(3)'	[0.7078]	[0.2366]
's^2'	[22.7571]	[7.0078]

APPENDIX 3

Unrestricted regime change

------- Before Regime Change ----------

'Coeff.'	'Post. mean'	'Post. std'
'C(0)'	[-23.5789]	[1.1645]
'C(1)'	[7.4562]	[0.7588]

's^2'	[0.2302]	[0.2762]

------- After Regime Change ----------

'Coeff.'	'Post. mean'	'Post. std'
'C(0)'	[2.7298]	[1.9614]
'C(1)'	[0.0074]	[0.1374]
's^2'	[14.7063]	[4.5556]
[]	[]	[]
'switch'	[2.0967]	[0.3012]

Connetcted regime change

'Coeff.'	'Post. mean'	'Post. std'
'Const'	[-16.0164]	[5.6841]
'C(1) Before'	[4.9024]	[2.3379]
'C(1) After'	[-0.0883]	[0.3328]
's^2'	[16.4120]	[5.5463]
'Switch'	[5.0249]	[3.1956]

APPENDIX 4

Bayesian algorithm for selecting the real GDP rate determinants

Coef.	Post.mean	Post.std
C(0)	-1.657	3.087
C(1)	-0.078	0.646
C(2)	0.241	0.652
C(3)	-0.094	0.588
C(4)	-0.444	0.757
C(5)	0.002	0.082
C(6)	-0.000	0.006
C(7)	1.659	2.538
C(8)	-0.004	0.020
C(9)	-0.038	0.040
C(10)	0.053	0.281
s^2	19.037	7.555

Variable Inclusion Probabilities

Coef.	Post.mean	Post.std
Tau(0)	0.521	0.500
Tau(1)	0.271	0.445
Tau(2)	0.302	0.459
Tau(3)	0.298	0.457
Tau(4)	0.426	0.494
Tau(5)	0.072	0.259
Tau(6)	0.015	0.120
Tau(7)	0.719	0.450
Tau(8)	0.052	0.223
Tau(9)	0.500	0.500
Tau(10)	0.213	0.410

------ Refined Regression Model ------

Coef.	Post.mean	Post.std
C(0)	-3.381	3.020
C(1)	0.235	0.112

C(2)	-0.984	0.402
C(3)	2.987	1.791
C(4)	-0.054	0.022
s^2	14.019	4.472

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